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Corrigendum to "Bayesian reduced rank regression in
econometrics" [J. Econometrics, 75 (1996) 121-146]

Sune Karlsson*

Örebro University

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*Department of Statistics, Örebro University Business School, 701 82 Örebro, Sweden. E-mail:
Sune.Karlsson@oru.se

Geweke (1996) studied the reduced rank regression model $\mathbf{Y} = \mathbf{X}\boldsymbol{\Theta} + \mathbf{Z}\mathbf{A} + \mathbf{E}$ where $\mathbf{Y}$ is an $n \times L$ matrix of dependent variables, $\mathbf{X}$ and $\mathbf{Z}$ contains p and k explanatory variables and $\boldsymbol{\Theta}$ and $\mathbf{A}$ are parameter matrices where $\boldsymbol{\Theta}$ is assumed to have reduced rank $q < \min(L, p)$. The rows of $\mathbf{E}$ are assumed to be independent normal with mean zero and variance matrix $\boldsymbol{\Sigma}$. Under the reduced rank assumption $\boldsymbol{\Theta}$ can be factored into $\boldsymbol{\Theta} = \boldsymbol{\Psi}\boldsymbol{\Phi}$ with $\boldsymbol{\Psi}$ a $p \times q$ matrix and $\boldsymbol{\Phi}$ a $q \times L$ matrix, both of rank q . To identify the model Geweke considers two normalizations, $\boldsymbol{\Phi} = (\mathbf{I}_q, \boldsymbol{\Phi}^*)$ with $\boldsymbol{\Psi}$ unrestricted (normalization 1) and $\boldsymbol{\Psi}' = (\mathbf{I}_q, \boldsymbol{\Psi}'^*)$ with $\boldsymbol{\Phi}$ unrestricted (normalization 2). Geweke provides full conditional posterior distributions for an informative prior where $\boldsymbol{\Sigma}$ is distributed as inverse Wishart with parameter matrix $\underline{\mathbf{S}}$ and $\underline{v}$ degrees of freedom, $iW(\underline{\mathbf{S}}, \underline{v})$, and the elements of $\mathbf{A}$, $(\boldsymbol{\Psi}, \boldsymbol{\Phi}^*)$ or $(\boldsymbol{\Phi}^*, \boldsymbol{\Psi})$ are independent normal with mean zero and variance $1/\tau^2$. Geweke also considers a flat, improper, prior on $\mathbf{A}$, $(\boldsymbol{\Psi}, \boldsymbol{\Phi}^*)$ or $(\boldsymbol{\Phi}^*, \boldsymbol{\Psi})$ which corresponds to setting $\tau = 0$ in the independent normals.

The full conditional posteriors for the improper prior are correctly stated in Geweke (1996) while the full conditional posteriors are incorrect in several cases for the proper informative prior. The incorrect expressions for the parameters of the posterior distributions have, unfortunately, been picked up in the literature and used to construct Gibbs samplers with incorrect stationary distributions. Geweke (2004) developed a method for checking the correctness of posterior simulators and detected problems with the Gibbs sampler coded up for the 1996 paper but failed to connect this with the incorrect expressions for the full conditional posteriors.

The mean of the full conditional posterior for $\boldsymbol{\Psi}$ in normalization 1 given in equation (11) of Geweke is incorrect. Let $\boldsymbol{\psi} = \text{vec}(\boldsymbol{\Psi})$, the correct full conditional posterior is normal, $\boldsymbol{\psi}|\mathbf{Y}, \mathbf{A}, \boldsymbol{\Phi}, \boldsymbol{\Sigma} \sim N(\bar{\boldsymbol{\psi}}, \mathbf{V}_{\boldsymbol{\psi}})$, for

$$\mathbf{V}_{\boldsymbol{\psi}} = (\tau^2 \mathbf{I} + \boldsymbol{\Phi} \boldsymbol{\Sigma}^{-1} \boldsymbol{\Phi}' \otimes \mathbf{X}' \mathbf{X})^{-1}, \quad \bar{\boldsymbol{\psi}} = \mathbf{V}_{\boldsymbol{\psi}} \text{vec}(\mathbf{X}' \mathbf{Y}^* \boldsymbol{\Sigma}^{-1} \boldsymbol{\Phi}') \quad (1)$$

where $\mathbf{Y}^* = \mathbf{Y} - \mathbf{Z}\mathbf{A}$.

The mean and variance of the full conditional posterior for Φ^* in normalization 1 given in equation (13) of Geweke are incorrect. Let $\phi^* = \text{vec}(\Phi^*)$, the correct full conditional posterior is normal, $\phi^* | \mathbf{Y}, \mathbf{A}, \Psi, \Sigma \sim N(\bar{\phi}^*, \mathbf{V}_{\phi^*})$, for

$$\mathbf{V}_{\phi^*} = [\tau^2 \mathbf{I} + \Sigma^{22} \otimes \Psi' \mathbf{X}' \mathbf{X} \Psi]^{-1}, \quad \bar{\phi}^* = \mathbf{V}_{\phi^*} \text{vec} [\Psi' \mathbf{X}' (\mathbf{Y}_1^* \Sigma^{12} - \mathbf{X} \Psi \Sigma^{12} + \mathbf{Y}_2^* \Sigma^{22})] \quad (2)$$

where $(\mathbf{Y}_1^*, \mathbf{Y}_2^*) = \mathbf{Y}^*$ partitions $\mathbf{Y}^*$ into $n \times q$ and $n \times (L - q)$ matrices and Σ^{12} and Σ^{22} are the upper right $q \times (L - q)$ and lower right $(L - q) \times (L - q)$ submatrices of Σ^{-1} .

The mean and variance of the full conditional posterior for Ψ^* in normalization 2 given in equation (15) of Geweke are incorrect. Let $\psi^* = \text{vec}(\Psi^*)$, the correct full conditional posterior is normal, $\psi^* | \mathbf{Y}, \mathbf{A}, \Phi, \Sigma \sim N(\bar{\psi}^*, \mathbf{V}_{\psi^*})$ for

$$\mathbf{V}_{\psi^*} = (\tau^2 \mathbf{I} + \Phi \Sigma^{-1} \Phi' \otimes \mathbf{X}_2' \mathbf{X}_2)^{-1}, \quad \bar{\psi}^* = \mathbf{V}_{\psi^*} \text{vec} (\mathbf{X}_2' (\mathbf{Y}^* - \mathbf{X}_1 \Phi) \Sigma^{-1} \Phi') \quad (3)$$

where $\mathbf{X}_1$ contains the first q columns of $\mathbf{X}$ and $\mathbf{X}_2$ the remaining columns

Additional details and derivations are provided in Karlsson (2012). The correct expressions for the improper prior in Geweke (1996) can be obtained as special cases by setting $\tau = 0$ in (1), (2) and (3).

References

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