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Type-2 Fuzzy Modeling and Control for Bilateral Teleoperation System with Dynamic Uncertainties and Time-varying Delays

Da Sun, Qianfang Liao and Hongliang Ren

Abstract – This paper develops data-driven Type-2 Takagi-Sugeno (T-S) fuzzy modeling and control for bilateral teleoperation with dynamic uncertainties and time-varying delays. The Type-2 T-S fuzzy model identified based on input-output data samples describes the nonlinear teleoperation system by a weighted sum of a group of linear local models, which offers a platform to design robust control algorithms by means of mature linear theories. The fuzzy-model-based four-channel control laws are proposed to guarantee the motion synchronization and enhance the operator's force perception for the environment when the time-varying delays and large dynamic uncertainties, especially the gravity of a heavy end effector of the slave, exist. Markov processes are applied to model the time delays. The stability of the closed-loop system is proved by using the Lyapunov-Krasovskii functions. All the conditions are expressed as linear matrix inequalities (LMIs). By using the Matlab LMI toolbox, the optimized control gains for each of the fuzzy rules are derived to achieve the optimal performance. Finally, experiments based on an experimental platform consisting of two haptic devices prove the superiority of the proposed strategy through comparison with previous work.

Index terms – Bilateral teleoperation, Type-2 T-S Fuzzy System, Interval time-varying delays, Robust control.

I. INTRODUCTION

Fuzzy logic model is effective to describe complex nonlinear systems for controller design because i) no exact mathematical system functions are required since fuzzy models are universal approximators and can be built based on input-output data samples, expert experience or both [1]-[3]; ii) fuzzy logic control has been proven to be a powerful tool to deal with the disturbances [4]. Two different fuzzy models, Mamdani [5] and Takagi-Sugeno (T-S) [6], are popular in researches and applications. Both of them are composed of IF-THEN rules with the same antecedent part but different consequents. Unlike Mamdani fuzzy model using fuzzy sets as local models, T-S fuzzy model using linear functions instead has the strengths to reflect the system dynamic properties more clearly, and requires less fuzzy rules [4]. In addition, the controller design for a nonlinear system can enjoy the well-developed theories of linear systems [4].

The traditional (Type-1) fuzzy model with crisp fuzzy memberships is limited in describing a system under the influence of uncertainties, and subsequently its controller design may provide degraded performance. In order to overcome this limitation, an extension work, called Type-2 fuzzy model [7]-[14], is proposed. In a Type-2 fuzzy set, the fuzzy membership of an element consists of primary and secondary grades that can be regarded as a Type-1 fuzzy set since secondary grades are used to characterize the possibilities of primary grades. As shown in Fig. 1, a general Type-2 fuzzy set as in part (a) has the secondary grades ranging from 0 to 1. When all the secondary grades are 1, it

becomes interval Type-2 fuzzy set [12]-[14] as shown in part (b), which is much more widely used because of its greatly reduced computational complexity. The increased fuzziness endows a Type-2 fuzzy set additional capability to describe the uncertainties.

Fig. 1. Type-2 fuzzy sets

The completed theories of Type-2 Mamdani fuzzy logic systems have been developed at the end of the last century [7]. Later, the introduction of Type-2 T-S fuzzy model was given [11], where the expression of interval Type-2 T-S fuzzy model has been stated that at least one of the following conditions should be satisfied: i) in the antecedents, interval Type-2 fuzzy sets are used to characterize the premise variables; ii) in the consequents, intervals instead of crisp numbers are used as the local model's coefficients. Several results [15]-[17] demonstrated that Type-2 T-S fuzzy systems improves the robustness in modeling and control performance when compared to its Type-1 counterparts.

This paper concerns the development of advanced fuzzy model based control algorithms for bilateral teleoperation systems. Fuzzy logic model is a good choice to describe the bilateral teleoperation system where the dynamic mathematic functions are unavailable. In the existing researches of controller designs for teleoperation systems, only a few methods utilizing fuzzy logic models have been found. Such as in [18]-[20], adaptive fuzzy control methods are proposed using Type-1 Mamdani fuzzy models to approximate the uncertain part of the systems. Compared to Mamdani fuzzy model, T-S fuzzy model can better describe the system dynamics. A T-S fuzzy model based controller is developed in [21] for Master-Slave bilateral teleoperation that using Type-1 T-S fuzzy model to describe the overall system for controller design. As introduced before, the uncertainties may not be fully handled by the Type-1 T-S fuzzy model. Moreover, assuming perfect gravity compensation is the main shortage of the controller design in [21] that is difficult to realize in real applications, especially for large-scale and complex robotic systems. To the best of author's knowledge, only one academic article [22] in this area using Type-2 fuzzy logic system is available that presenting a bilateral teleoperation controller design based on a Type-2 fuzzy wavelet neural network. The Type-2 fuzzy system used in [22] has increased fuzziness in antecedents but remaining crisp coefficients in consequents, which may not be able to

properly describe the uncertainties since only antecedent part is Type-2. Also, only simulation studies are provided in [22].

On the other hand, a well-designed bilateral teleoperation system should be able to achieve reasonable position synchronization, and, more importantly, can accurately feed the environmental forces back to the master side in order to allow the operator to have a good perception on the remote object. Numerous papers propose designs to achieve the accurate position tracking of the bilateral teleoperation, such as [18], [29]-[31]. While the force tracking and the perception of the operators on the environment are seldom considered in the existing research articles. Although force control methods are designed based on Disturbance Observers in linear teleoperation systems [32]-[33], their performance in multi-DOF nonlinear teleoperation systems are still questionable. Besides, in teleoperation control of the complex robots, the existence of large uncertainties and time-varying delays will jeopardize the system stabilities and, furthermore, degrade the position tracking and the operator's perception on the remote environment. Therefore, effective method to deal with dynamic uncertainties and time delays needs to be studied to provide transparent force feedback and position tracking of the bilateral teleoperation systems.

This paper proposes a data-driven Type-2 T-S fuzzy modeling and control algorithm for the bilateral teleoperation system. The steps to construct an interval Type-2 T-S fuzzy model based on input-output data are given that possessing increased fuzziness in both antecedents and consequents to guarantee the sufficient capability to compensate the uncertainties. Subsequently a nonlinear bilateral teleoperation system with unknown dynamics can be regarded as a combination of multiple linear systems via the proposed fuzzy rules. Compared with previous work, the proposed approach has superiority on compensation for large dynamic uncertainties. In this way, the control laws for linear systems can also be freely utilized in the nonlinear teleoperation system and can achieve better performance than some control algorithms for nonlinear systems. Four-channel control laws using Type-2 fuzzy gains are proposed to provide motion synchronization and simultaneously enhance the operator's force perception on the remote environment. The time-varying delays with upper and lower bounds are considered in this paper, and Markov processes are used to model these random network induced delays. Lyapunov-Krasovskii functions are used to analyze the system stability with dynamic uncertainties. The configuration of the control gains for each of the fuzzy rules are expressed by the Linear Matrix Inequalities (LMIs) and can be optimized using the Matlab LMI toolbox. The superiority of the proposed approach is demonstrated by a bilateral experimental platform consisting two haptic devices, in which the slave has a heavy end effector that enlarges the gravity uncertainties. A number of experimental comparisons with previous work are also demonstrated.

II. TYPE-2 T-S FUZZY MODELING

The dynamics a Master-Slave bilateral teleoperation system are described as follows:

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) + f_i\dot{q}_i + f_{ci}(\dot{q}_i) + F_i^d = \tau_i + \tau_j \quad (1)$$

where $i = m, s$ represents master and slave, and $j = h, e$ denotes human and environment. $\ddot{q}_i, \dot{q}_i, q_i \in R^n$ ($q_i \in R^n$)

stand for the joint acceleration, velocity and position signals, respectively. $M_i(q_i) \in R^{n \times n}$ and $C_i(q_i, \dot{q}_i) \in R^{n \times n}$ are the inertia matrices and Coriolis/centrifugal effects, respectively. $g_i(q_i) \in R^n$ is the gravitational force. $f_i\dot{q}_i$ and $f_{ci}(\dot{q}_i)$ denote the viscous friction and Coulomb friction, respectively. F_i^d is the unknown disturbance. τ_i is the control signal, and τ_j is the external human or environmental torques that $\tau_j = \tau_j^* + \Delta\tau_j$, where τ_j^* is the measured torque and $\Delta\tau_j$ is the measurement error.

This paper utilizes Type-2 T-S fuzzy models to represent the bilateral teleoperation system for controller design. T-S fuzzy model describes a global nonlinear system using a group of linear local models which are blended by fuzzy membership functions. It has been proved to be a universal approximator to any smooth nonlinear functions to any degree of accuracy in any convex compact area [2]-[3]. As an extension work of Type-1 T-S fuzzy model, the fuzzy rules of an interval Type-2 T-S fuzzy model [11] can be expressed as:

Rule l : IF $x(k)$ is $\tilde{C}^l$, THEN

$$\tilde{y}^l(k) = \tilde{a}_0^l + \tilde{a}_1^l x_1(k) + \tilde{a}_2^l x_2(k) + \dots + \tilde{a}_n^l x_n(k) \quad (2)$$

where $l = 1, \dots, c$, c is the number of fuzzy rules; $x(k) \in R^n$ is the input vector of k th sampling time $x(k) = [x_1(k) \ x_2(k) \ \dots \ x_n(k)]^T$; $\tilde{C}^l$ is the l th interval Type-2 fuzzy set [5] where the fuzzy membership of $x(k)$ is an interval denoted as $\tilde{\mu}^l(x(k)) = [\underline{\mu}^l(x(k)), \overline{\mu}^l(x(k))]$, $\underline{\mu}^l(x(k))$ and $\overline{\mu}^l(x(k))$ are lower and upper bounds respectively, and $0 \leq \underline{\mu}^l(x(k)) \leq \overline{\mu}^l(x(k)) \leq 1$; The coefficients of local model are also intervals as $\tilde{a}_r^l = [\underline{a}_r^l, \overline{a}_r^l]$, $r = 0, 1, \dots, n$. And the local output $\tilde{y}^l(k) = [\underline{y}^l(k), \overline{y}^l(k)]$ is derived by:

$$\begin{cases} \underline{y}^l(k) = \underline{a}_0^l + \underline{a}_1^l x_1(k) + \underline{a}_2^l x_2(k) + \dots + \underline{a}_n^l x_n(k) \\ \overline{y}^l(k) = \overline{a}_0^l + \overline{a}_1^l x_1(k) + \overline{a}_2^l x_2(k) + \dots + \overline{a}_n^l x_n(k) \end{cases} \quad (3)$$

Blending the c Type-2 fuzzy rules creates a type-reduced set which is an interval denoted as $\tilde{y}(k) = [\underline{y}(k), \overline{y}(k)]$. Karnik-Mendel algorithm [8] is a frequently used method to derive $\underline{y}(k)$ and $\overline{y}(k)$. However, Karnik-Mendel algorithm requires iterative calculations which may be time consuming and cause unwanted time delay. In this paper, the following method is selected for simplification [16]-[17]:

$$\begin{cases} \underline{y}(k) = \frac{\sum_{l=1}^c \underline{\mu}^l(x(k)) \underline{y}^l(k)}{\sum_{l=1}^c \underline{\mu}^l(x(k))} \\ \overline{y}(k) = \frac{\sum_{l=1}^c \overline{\mu}^l(x(k)) \overline{y}^l(k)}{\sum_{l=1}^c \overline{\mu}^l(x(k))} \end{cases} \quad (4)$$

Then defuzzifying the type-reduced set $\tilde{y}(k)$ gives the crisp output $y(k)$:

$$y(k) = \frac{\underline{y}(k) + \overline{y}(k)}{2} \quad (5)$$

A Type-2 T-S fuzzy model can be constructed based on data samples, human experience or a combination of both. Collect the input-output pairs from the original system as $z(k) = [x^T(k) \ y(k)]^T$, where $z(k) \in R^{n+1}$, $k = 1, \dots, N$, N is the number of samples. Gustafson-Kessel (G-K) clustering algorithm [23] is employed to form c fuzzy clusters based on the data samples in which c fuzzy cluster centers are located for fuzzy membership calculations. Correspondingly, the system is characterized by c fuzzy rules, where the

coefficients of the local models are identified by weighted least square method. The steps are given as follows:

1). Set the initial value of cluster number $c = c_0$, where c_0 is a small integer; and set a tolerance $\varepsilon > 0$ for the root-mean-square-error (RMSE) of the identified model.

2). Use G-K algorithm to determine c fuzzy cluster centers from data samples, denoted as $z_c^l = [(x_c^l)^T \ y_c^l]^T$, $l = 1, \dots, c$, where $z_c^l \in R^{n+1}$, $x_c^l \in R^n$, $x_c^l = [x_{c,1}^l(k) \ x_{c,2}^l(k) \ \dots \ x_{c,n}^l(k)]^T$. Subsequently a fuzzy partition matrix $U = [\mu^l(z(k))]_{c \times N}$ can be obtained where $\mu^l(z(k))$ is the crisp fuzzy membership of $z(k)$ in the l th fuzzy cluster that is calculated according to the distances between $z(k)$ and the c centers, and satisfies $0 \leq \mu^l(z(k)) \leq 1$ and $\sum_{l=1}^c \mu^l(z(k)) = 1$.

3). For each cluster, select N_1^l ($N_1^l \geq 3$) samples with the Euclidean distance of input between each other not larger than a given small value $\varepsilon_1 > 0$ as:

$$\|x(a) - x(b)\| \leq \varepsilon_1, \quad a, b = 1, \dots, N_1^l$$

The blurring radiuses of fuzzy membership and output of this cluster, denoted by $\Delta\mu^l$ and Δy^l , respectively, can be determined by $\Delta\mu^l = \frac{\max\{|\mu^l(z(a)) - \mu^l(z(b))|\}}{2}$, $\Delta y^l = \frac{\max\{|y(a) - y(b)|\}}{2}$, $a, b = 1, \dots, N_1^l$. Note that $\Delta\mu^l$ and Δy^l can also be determined by expert experience. Denote $\Delta y = \max\{\Delta y^l\}$, $l = 1, \dots, c$.

4). For each cluster, a crisp fuzzy membership $\mu^l(z(k))$ can be extended to an interval $\tilde{\mu}^l(z(k)) = [\underline{\mu}^l(z(k)), \bar{\mu}^l(z(k))]$:

$$\begin{cases} \underline{\mu}^l(z(k)) = \max\{0, \mu^l(z(k)) - \Delta\mu^l\} \\ \bar{\mu}^l(z(k)) = \min\{\mu^l(z(k)) + \Delta\mu^l, 1\} \end{cases} \quad (6)$$

5). Each sample $z(k)$ can be turn into two, $\underline{z}(k)$ and $\bar{z}(k)$, by the following equations:

$$\begin{cases} \underline{z}(k) = [x^T(k) \ y(k) - \Delta y]^T \\ \bar{z}(k) = [x^T(k) \ y(k) + \Delta y]^T \end{cases} \quad (7)$$

For each fuzzy rule, two linear polynomials as in (3) with the coefficients denoted as $\underline{a}^l = [\underline{a}_0^l \ \dots \ \underline{a}_n^l]^T$ and $\bar{a}^l = [\bar{a}_0^l \ \dots \ \bar{a}_n^l]^T$, respectively, can be identified by weighted least square method where the crisp fuzzy memberships are used as weights:

$$\begin{cases} \underline{a}^l = (X^T W^l X)^{-1} X^T W^l Y \\ \bar{a}^l = (X^T W^l X)^{-1} X^T W^l \bar{Y} \end{cases} \quad (8)$$

$$\text{where } X = \begin{bmatrix} 1 & x_1(1) & \dots & x_n(1) \\ 1 & x_1(2) & \dots & x_n(2) \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_1(N) & \dots & x_n(N) \end{bmatrix}_{N \times (n+1)}, \quad Y =$$

$$\begin{bmatrix} y(1) - \Delta y & y(2) - \Delta y & \dots & y(N) - \Delta y \end{bmatrix}^T, \quad \bar{Y} = \begin{bmatrix} y(1) + \Delta y & y(2) + \Delta y & \dots & y(N) + \Delta y \end{bmatrix}^T \quad \text{and} \quad W^l = \text{diag}[\mu^l(z(1)) \ \mu^l(z(2)) \ \dots \ \mu^l(z(N))].$$

6). Based on the results obtained by (6) and (8), using (3)-(5) can calculate the outputs of the model according to the inputs of the data samples, subsequently the RMSE can be derived by comparing them to the outputs of the data sample. If the RMSE is not larger than the tolerance ε , then the identified result is satisfactory and the modeling is complete. Otherwise, let $c = c + 1$, and repeat step 2)-5).

When given a new input $x(k)$, a crisp fuzzy membership can be calculated by:

$$\mu^l(x(k)) = \begin{cases} 0, & \text{If } \forall D^2(x(k), x_c^l) = 0, v = 1, \dots, c, v \neq l \\ \frac{1}{\sum_{v=1}^c \frac{D^2(x(k), x_c^l)}{D^2(x(k), x_c^v)}}, & \text{if all } D^2(x(k), x_c^v) \neq 0, v = 1, \dots, c \\ 1, & \text{if } D^2(x(k), x_c^l) = 0 \end{cases} \quad (9)$$

where $D^2(x(k), x_c^l) = \|x(k) - x_c^l\|$, $l = 1, \dots, c$, are the Euclidean distances between $x(k)$ and the fuzzy cluster centers. The lower and the upper bounds of the interval Type-2 fuzzy membership for $x(k)$ in l th fuzzy cluster are:

$$\begin{cases} \underline{\mu}^l(x(k)) = \max\{0, \mu^l(x(k)) - \Delta\mu^l\} \\ \bar{\mu}^l(x(k)) = \min\{\mu^l(x(k)) + \Delta\mu^l, 1\} \end{cases} \quad (10)$$

Then the output $y(k)$ of the Type-2 fuzzy model is derived by (3)-(5) where the coefficients of local models are identified by (8). Using Type-2 T-S fuzzy model to describe the bilateral teleoperation is able to greatly facilitate controller design since the nonlinear system in (1) can be represented by a weighted sum of a group of linear models where the coefficients are intervals instead of crisp numbers such that they possess additional power to describe uncertainties. For a Master or a Slave with degree of freedom as n^f , collect the joint positions $q_i(k) \in R^{n^f}$, joint velocities $v_i(k) \in R^{n^f}$, and torques $(\tau_i(k) + \tau_j^*(k)) \in R^{n^f}$, $k = 1, \dots, N$, to form the data samples as $z_i(k) = [x^T(k) \ (\tau_i(k) + \tau_j^*(k))^T]^T$, where $x_i(k) = [ac_i^T(k) \ v_i^T(k) \ q_i^T(k)]^T$. $ac_i(k) = \frac{v_i(k) - v_i(k-1)}{\Delta T}$ can represent the joint accelerations when the sampling period ΔT is sufficiently small. The Type-2 fuzzy rules identified from $z_i(k)$, $k = 1, \dots, N$, can be expressed as:

Rule l : IF $x_i(k)$ is $\tilde{C}^l$, THEN

$$\tilde{M}_i^l ac_i(k) + \tilde{C}_i^l v_i(k) + \tilde{D}_i^l q_i(k) + \tilde{E}_i^l = \tilde{\tau}_i^l(k) + \tau_j^*(k) \quad (11)$$

where the centre of $\tilde{C}^l$ is denoted as $z_{i,c}^l$, and its blurring radius is $\Delta\mu_i^l$. $\tilde{M}_i^l = [\underline{M}_i^l, \bar{M}_i^l]$, $\tilde{C}_i^l = [\underline{C}_i^l, \bar{C}_i^l]$, $\tilde{D}_i^l = [\underline{D}_i^l, \bar{D}_i^l]$ and $\tilde{E}_i^l = [\underline{E}_i^l, \bar{E}_i^l]$ that $\underline{M}_i^l, \bar{M}_i^l, \underline{C}_i^l, \bar{C}_i^l, \underline{D}_i^l, \bar{D}_i^l \in R^{n^f \times n^f}$ are diagonal matrices while $\underline{E}_i^l, \bar{E}_i^l \in R^{n^f}$ are vectors. $\underline{M}_i^l, \bar{M}_i^l, \underline{C}_i^l, \bar{C}_i^l, \underline{D}_i^l, \bar{D}_i^l$ and $\underline{E}_i^l, \bar{E}_i^l$ actually are the upper and lower boundaries of $M_i(q_i)$, $C_i(q_i, \dot{q}_i)$ and related dynamic uncertainties including gravity. $\tilde{\tau}_i^l(k) = [\underline{\tau}_i^l(k), \bar{\tau}_i^l(k)]$ determines the control laws for these upper and lower boundaries. The Type-2 T-S fuzzy model as in (11) for each of the fuzzy rule can be expressed by the following equations:

$$\underline{M}_i^l \ddot{q}_i + \underline{C}_i^l \dot{q}_i + \underline{D}_i^l q_i + \underline{E}_i^l = \underline{\tau}_i^l + \tau_j^* \quad (12)$$

$$\bar{M}_i^l \ddot{q}_i + \bar{C}_i^l \dot{q}_i + \bar{D}_i^l q_i + \bar{E}_i^l = \bar{\tau}_i^l + \tau_j^* \quad (13)$$

$$\begin{aligned} \text{Treating } M_i &= \frac{1}{2}(\bar{M}_i + \underline{M}_i) = \frac{\sum_{l=1}^c \underline{\mu}^l(x_i(k)) \underline{M}_i^l}{2 \sum_{l=1}^c \underline{\mu}^l(x_i(k))} + \frac{\sum_{l=1}^c \bar{\mu}^l(x_i(k)) \bar{M}_i^l}{2 \sum_{l=1}^c \bar{\mu}^l(x_i(k))}, \\ C_i &= \frac{1}{2}(\bar{C}_i + \underline{C}_i) = \frac{\sum_{l=1}^c \underline{\mu}^l(x_i(k)) \underline{C}_i^l}{2 \sum_{l=1}^c \underline{\mu}^l(x_i(k))} + \frac{\sum_{l=1}^c \bar{\mu}^l(x_i(k)) \bar{C}_i^l}{2 \sum_{l=1}^c \bar{\mu}^l(x_i(k))}, \quad D_i = \frac{1}{2}(\bar{D}_i + \underline{D}_i) \\ &= \frac{\sum_{l=1}^c \underline{\mu}^l(x_i(k)) \underline{D}_i^l}{2 \sum_{l=1}^c \underline{\mu}^l(x_i(k))} + \frac{\sum_{l=1}^c \bar{\mu}^l(x_i(k)) \bar{D}_i^l}{2 \sum_{l=1}^c \bar{\mu}^l(x_i(k))}, \quad E_i = \frac{1}{2}(\bar{E}_i + \underline{E}_i) = \\ &= \frac{\sum_{l=1}^c \underline{\mu}^l(x_i(k)) \underline{E}_i^l}{2 \sum_{l=1}^c \underline{\mu}^l(x_i(k))} + \frac{\sum_{l=1}^c \bar{\mu}^l(x_i(k)) \bar{E}_i^l}{2 \sum_{l=1}^c \bar{\mu}^l(x_i(k))}, \quad \tau_i = \frac{\sum_{l=1}^c \underline{\mu}^l(x_i(k)) \underline{\tau}_i^l(k)}{2 \sum_{l=1}^c \underline{\mu}^l(x_i(k))} + \frac{\sum_{l=1}^c \bar{\mu}^l(x_i(k)) \bar{\tau}_i^l(k)}{2 \sum_{l=1}^c \bar{\mu}^l(x_i(k))}. \end{aligned} \quad (12) \text{ and } (13) \text{ can be simplified as}$$

$$(M_i + \Delta M_i)\ddot{q}_i + (C_i + \Delta C_i)\dot{q}_i + (D_i + \Delta D_i)q_i + (E_i + \Delta E_i) = \tau_i + \tau_i^* \quad (14)$$

where $\Delta M_i = \frac{1}{2}\lambda(\bar{M}_i - \underline{M}_i)$, $\Delta C_i = \frac{1}{2}\lambda(\bar{C}_i - \underline{C}_i)$, $\Delta D_i = \frac{1}{2}\lambda(\bar{D}_i - \underline{D}_i)$, $\Delta E_i = \frac{1}{2}\lambda(\bar{E}_i - \underline{E}_i)$. λ is an unknown variable varying between -1 and 1.

Remark 1: By properly choosing the varying ranges of fuzzy membership grades $\Delta\mu^l$ and outputs Δy during the modeling process, the identified lower and upper bounds of the coefficients in the designed Type-2 fuzzy dynamic model (12)-(13) can cover the situations with inexactness. As a result, ΔM_i , ΔC_i , ΔD_i and ΔE_i can properly describe the degree of dynamic uncertainty. To maintain the absolutely stability of the system, the terms ΔM_i , ΔC_i , ΔD_i and ΔE_i are concerned in the control laws design and stability analysis.

III. CONTROL ALGORITHMS

Remark 2: Inspired by [19], we assume feedforward time delay $T_1(t)$ and the feedback time delay $T_2(t)$ to be mode-dependent time-varying and are governed by two Markov processes, i.e., $\varpi_1(t)$ and $\varpi_2(t)$, respectively, which are two independent continuous-time discrete-state Markov processes that take values in a finite set $\mathbb{Q} = \{1, 2, \dots, n\}$ with a transition probability matrix given by

$$P(\varpi_\varrho(t + \Delta) = h | \varpi_\varrho(t) = h') = \begin{cases} \pi_{h'h}\Delta + o(\Delta), & h \neq h' \\ 1 + \pi_{h'h'}\Delta + o(\Delta), & h = h' \end{cases}$$

where $\varrho = 1, 2$, $\Delta > 0$, $\lim_{\Delta \rightarrow 0} o(\Delta)/\Delta = 0$. $\pi_{h'h} \geq 0$ ($h', h \in \mathbb{Q}$, $h \neq h'$) represent the transition rate from mode h' to h . $\pi_{h'h'} = -\sum_{h=1, h \neq h'}^n \pi_{h'h}$, for all $h' \in \mathbb{Q}$.

Remark 3: The differentials of $T_1(t)$ and $T_2(t)$ are bounded by positive constants $\bar{d}_1$ and $\bar{d}_2$, respectively. That is, $0 \leq |\dot{T}_{1,2}(t)| \leq \bar{d}_{1,2}$. The values of $T_{1,2}(t)$ also have upper and lower bounds, i.e., $\underline{T}_{1,2} \leq T_{1,2}(t) \leq \bar{T}_{1,2}$.

Remark 4: The proposed Type-2 fuzzy model can be applied to arbitrary control laws. The control laws introduced later (four-channel teleoperation with Type-2 fuzzy gains) makes a clear example that even without complex adaptive control method, the system can still have highly-developed performance. The priority of this paper is to provide reasonable force reflection and position tracking of bilateral teleoperation with system uncertainties under time-varying delay conditions.

We define the position and the measured torque tracking errors as follows:

$$e_m(t) = q_m(t) - q_s(t - T_2(t)), \quad e_s(t) = q_s(t) - q_m(t - T_1(t)) \quad (15)$$

$$e_{tm}(t) = \tau_h^*(t) - \tau_e^*(t - T_2(t)), \quad e_{ts}(t) = \tau_h^*(t - T_1(t)) - \tau_e^*(t) \quad (16)$$

The torque tracking errors are expected to follow a decaying function of time, restricted by a predefined boundary, and converge to the neighbor of zero at the steady state. That is,

$$\begin{cases} -p\rho_i(t) < e_{ti}(t) < \rho_i(t), & \text{if } e_{ti}(0) \geq 0 \\ -\rho_i(t) < e_{ti}(t) < p\rho_i(t), & \text{if } e_{ti}(0) \leq 0 \end{cases} \quad (17)$$

where p is a constant that $0 < p < 1$, and $\rho_i(t)$ is the decaying function of time.

$$\rho_i(t) = (\rho_{i0} - \rho_{i\infty})e^{-\iota t} + \rho_{i\infty} \quad (18)$$

where ρ_{i0} , $\rho_{i\infty}$ and ι are positive constants. ρ_{i0} is the initial value of the function ρ_i . $\rho_{i\infty} = \lim_{t \rightarrow \infty} \rho_i(t)$ is the predefined final constraint of $\rho_i(t)$ to suppress the torque tracking errors at the steady state. ι is the convergence velocity. Based on the position tracking errors in (15), new variables $r_i(t)$ are defined:

$$r_i(t) = \dot{q}_i(t) + \Lambda_i e_i(t) \quad (19)$$

where Λ_i is a positive diagonal matrix. Define

$$\epsilon(e_{ti}(t)) = \hat{A}\left(\frac{e_{ti}(t)}{\rho_i(t)}\right) + e_{ti}(t) \quad (20)$$

where $\hat{A}(\cdot)$ is a smooth, strictly increasing function shown as

$$\hat{A}\left(\frac{e_{ti}(t)}{\rho_i(t)}\right) = \begin{cases} \ln \frac{\frac{e_{ti}(t)}{\rho_i(t)} + p}{p - \frac{e_{ti}(t)}{\rho_i(t)}}, & \text{if } e_{ti}(0) \geq 0 \\ \ln \frac{p - \frac{e_{ti}(t)}{\rho_i(t)}}{\frac{e_{ti}(t)}{\rho_i(t)}}, & \text{if } e_{ti}(0) < 0 \end{cases} \quad (21)$$

The control laws for the master and the slave are designed as:

$$\tau_m(t) = -k_1 r_m(t) + \kappa_m \epsilon(e_{tm}(t)) - M_m \Lambda_m (\dot{q}_m(t) - \dot{q}_s(t - T_2(t))) + C_m \dot{q}_m(t) + D_m q_m + E_m - \tau_h^*(t) \quad (22)$$

$$\tau_s(t) = -k_2 r_s(t) + \kappa_s \epsilon(e_{ts}(t)) - M_s \Lambda_s (\dot{q}_s(t) - \dot{q}_m(t - T_1(t))) + C_s \dot{q}_s(t) + D_s q_s + E_s - \tau_e^*(t) \quad (23)$$

k_{1-4} , κ_i are the diagonal matrices relating the designed fuzzy rules, where, for Type-2 fuzzy model, $k_1 = \frac{\sum_{l=1}^c \mu^l(x_m(k)) \bar{k}_1^l}{2 \sum_{l=1}^c \mu^l(x_m(k))} + \frac{\sum_{l=1}^c \bar{\mu}^l(x_m(k)) \bar{k}_1^l}{2 \sum_{l=1}^c \bar{\mu}^l(x_m(k))}$, $k_2 = \frac{\sum_{l=1}^c \mu^l(x_s(k)) \bar{k}_2^l}{2 \sum_{l=1}^c \mu^l(x_s(k))} + \frac{\sum_{l=1}^c \bar{\mu}^l(x_s(k)) \bar{k}_2^l}{2 \sum_{l=1}^c \bar{\mu}^l(x_s(k))}$, $\kappa_m = \frac{\sum_{l=1}^c \mu^l(x_m(k)) \bar{\kappa}_m^l}{2 \sum_{l=1}^c \mu^l(x_m(k))} + \frac{\sum_{l=1}^c \bar{\mu}^l(x_m(k)) \bar{\kappa}_m^l}{2 \sum_{l=1}^c \bar{\mu}^l(x_m(k))}$, $\kappa_s = \frac{\sum_{l=1}^c \mu^l(x_s(k)) \bar{\kappa}_s^l}{2 \sum_{l=1}^c \mu^l(x_s(k))} + \frac{\sum_{l=1}^c \bar{\mu}^l(x_s(k)) \bar{\kappa}_s^l}{2 \sum_{l=1}^c \bar{\mu}^l(x_s(k))}$. Adding (22) and (23) into (14), the dynamics models can finally be expressed as:

$$\dot{r}_m(t) = -\frac{k_1}{M_m} r_m(t) + \frac{\kappa_m}{M_m} \epsilon(e_{tm}(t)) + \dot{T}_2(t) \Lambda_m \dot{q}_s(t - T_2(t)) - \frac{\Delta M_m}{M_m} \ddot{q}_m - \frac{\Delta C_m}{M_m} \dot{q}_m - \frac{\Delta D_m}{M_m} q_m - \frac{\Delta E_m}{M_m} \quad (24)$$

$$\dot{r}_s(t) = -\frac{k_2}{M_s} r_s(t) + \frac{\kappa_s}{M_s} \epsilon(e_{ts}(t)) + \dot{T}_1(t) \Lambda_s \dot{q}_m(t - T_1(t)) - \frac{\Delta M_s}{M_s} \ddot{q}_s - \frac{\Delta C_s}{M_s} \dot{q}_s - \frac{\Delta D_s}{M_s} q_s - \frac{\Delta E_s}{M_s} \quad (25)$$

Based on the above equations, the overall close loop system can be expressed as:

$$\begin{bmatrix} \dot{e}_m(t) \\ \dot{e}_s(t) \\ \dot{r}_m(t) \\ \dot{r}_s(t) \end{bmatrix} = \begin{bmatrix} -\Lambda_m & 0 & I & 0 \\ 0 & -\Lambda_s & 0 & I \\ 0 & 0 & -\frac{k_1}{M_m} & 0 \\ 0 & 0 & 0 & -\frac{k_2}{M_s} \end{bmatrix} \begin{bmatrix} e_m(t) \\ e_s(t) \\ r_m(t) \\ r_s(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \mathfrak{S}_2 \Lambda_s & 0 & -\mathfrak{S}_2 I \\ \mathfrak{S}_1 \Lambda_m & 0 & -\mathfrak{S}_1 I & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_m(t - T_1(t)) \\ e_s(t - T_2(t)) \\ r_m(t - T_1(t)) \\ r_s(t - T_2(t)) \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\kappa_m}{M_m} & 0 \\ 0 & 0 & 0 & \frac{\kappa_s}{M_s} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \epsilon(e_{tm}(t)) \\ \epsilon(e_{ts}(t)) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & \dot{T}_2(t)M_m\Lambda_m \\ 0 & \dot{T}_1(t)M_s\Lambda_s \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{q}_m(t-T_1(t)) \\ \dot{q}_s(t-T_2(t)) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \mathcal{G}_m(t) \\ \mathcal{G}_s(t) \end{bmatrix} \quad (26)$$

where $\mathfrak{S}_1 = 1 - \dot{T}_1$ and $\mathfrak{S}_2 = 1 - \dot{T}_2$. $\mathcal{G}_m = -\frac{\Delta M_m}{M_m}\ddot{q}_m - \frac{\Delta C_m}{M_m}\dot{q}_m - \frac{\Delta D_m}{M_m}q_m - \frac{\Delta E_m}{M_m}$, $\mathcal{G}_s = -\frac{\Delta M_s}{M_s}\ddot{q}_s - \frac{\Delta C_s}{M_s}\dot{q}_s - \frac{\Delta D_s}{M_s}q_s - \frac{\Delta E_s}{M_s}$. Then the system in (26) can be decoupled into two subsystems with the X_1 subsystem:

$$\dot{r} = A_{11}r + A_{12}\epsilon + A_{13}\dot{q}(t-T) + A_{14}\ddot{q} + A_{15}\dot{q} + A_{16}q + A_{17} \quad (27)$$

where $A_{11} = \begin{bmatrix} -\frac{k_1}{M_m} & 0 \\ 0 & -\frac{k_3}{M_s} \end{bmatrix}$, $A_{12} = \begin{bmatrix} \frac{\kappa_m}{M_m} & 0 \\ 0 & \frac{\kappa_s}{M_s} \end{bmatrix}$, $A_{13} = \begin{bmatrix} 0 & \dot{T}_2(t)\Lambda_m \\ \dot{T}_1(t)\Lambda_s & 0 \end{bmatrix}$, $A_{14} = \begin{bmatrix} -\frac{\Delta M_m}{M_m} & 0 \\ 0 & -\frac{\Delta M_s}{M_s} \end{bmatrix}$, $A_{15} = \begin{bmatrix} -\frac{\Delta C_m}{M_m} & 0 \\ 0 & -\frac{\Delta C_s}{M_s} \end{bmatrix}$, $A_{16} = \begin{bmatrix} -\frac{\Delta D_m}{M_m} & 0 \\ 0 & -\frac{\Delta D_s}{M_s} \end{bmatrix}$, $A_{17} = \begin{bmatrix} -\frac{\Delta E_m}{M_m} \\ -\frac{\Delta E_s}{M_s} \end{bmatrix}$. $\dot{r} = [\dot{r}_m(t), \dot{r}_s(t)]^T$, $r = [r_m(t), r_s(t)]^T$, $\epsilon = [\epsilon(e_{tm}(t)), \epsilon(e_{ts}(t))]^T$, $\dot{q} = [\dot{q}_m(t), \dot{q}_s(t)]^T$, $\dot{q}(t-T) = [\dot{q}_m(t-T_1(t)), \dot{q}_s(t-T_2(t))]^T$, $\ddot{q} = [\ddot{q}_m(t), \ddot{q}_s(t)]^T$, $q = [q_m(t), q_s(t)]^T$.

and the X_2 subsystem:

$$\dot{e} = A_{21}e + A_{22}e(t-T) + A_{23}r + A_{24}r(t-T) \quad (28)$$

where $A_{21} = \begin{bmatrix} -\Lambda_m & 0 \\ 0 & -\Lambda_s \end{bmatrix}$, $A_{22} = \begin{bmatrix} 0 & (1-\dot{T}_2(t))\Lambda_s \\ (1-\dot{T}_1(t))\Lambda_m & 0 \end{bmatrix}$, $A_{23} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$, $A_{24} = \begin{bmatrix} 0 & -(1-\dot{T}_2(t))I \\ -(1-\dot{T}_1(t))I & 0 \end{bmatrix}$, $\dot{e} = [\dot{e}_m(t), \dot{e}_s(t)]^T$, $e = [e_m(t), e_s(t)]^T$, $r = [r_m(t), r_s(t)]^T$, $r(t-T) = [r_m(t-T_1(t)), r_s(t-T_2(t))]^T$. X_1 subsystem and X_2 subsystem represent the designed control system and the tracking errors convergence, respectively. Guaranteeing stability of the X_1 and X_2 subsystems determines the overall system stability and high motion synchronization.

IV. PERFORMANCE ANALYSIS

4.1. X_1 subsystem stability

$$\mathbf{e} = \begin{bmatrix} e' \\ \dot{e}' \\ \ddot{e}' \\ r' \\ \epsilon' \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} q + \begin{bmatrix} 0 & 0 \\ 1 & -1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \dot{q} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & -1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \ddot{q} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & -1 \\ 0 & 0 \end{bmatrix} r + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & -1 \end{bmatrix} \epsilon = X_1q + X_2\dot{q} + X_3\ddot{q} + X_4r + X_5\epsilon \quad (29)$$

(29) is the defined output where $e'(t) = q_m(t) - q_s(t)$, $r'(t) = r_m(t) - r_s(t)$, $\epsilon'(t) = \epsilon_m(t) - \epsilon_s(t)$. Our goal is to minimize $\mathbf{e}$, which contains the position, velocity, acceleration and torque tracking errors, by finding proper position and torque control gains k_{1-2} and κ_i such that X_1

subsystem is stable and the following H_∞ performance requirement is achieved.

$$\int_0^\infty \mathbf{e}^T(\eta)\mathbf{e}(\eta)d\eta < Y^2 \int_0^\infty \epsilon^T(\eta)\epsilon(\eta) + \dot{q}^T(\eta)\dot{q}(\eta)d\eta \quad (30)$$

According to (30), Y is the performance index of the system. More exactly smaller Y requires smaller $\mathbf{e}$, and the tracking errors can then be minimized. Consequently, we are trying to find the optimized control gains k_{1-2} and κ_i to satisfy the smallest Y .

Lemma [24]: For any constant matrix $M \in \mathbb{R}^{n \times n}$, $M = M^T > 0$, and $\beta \leq \eta \leq \alpha$, the following inequalities hold:

$$(\alpha - \beta) \int_\beta^\alpha \dot{x}^T(\eta)M\dot{x}(\eta)d\eta \geq (\int_\beta^\alpha \dot{x}(\eta)d\eta)^T M \int_\beta^\alpha \dot{x}(\eta)d\eta$$

$$\frac{(\alpha - \beta)^2}{2} \int_\beta^\alpha \int_\xi^\alpha \dot{x}^T(\eta)M\dot{x}(\eta)d\eta d\xi \geq (\int_\beta^\alpha \int_\xi^\alpha \dot{x}(\eta)d\eta d\xi)^T M \int_\beta^\alpha \int_\xi^\alpha \dot{x}(\eta)d\eta d\xi$$

Proof: To prove the stability of the proposed system in (30), consider the following Lyapunov-Krasovskii functional as: $V = V_1 + V_2 + V_3 + V_4$, where

$$V_1 = r_m^T P_m(\omega_2)r_m + r_s^T P_s(\omega_1)r_s \quad (31)$$

$$V_2 = \int_{t-T_1}^t r_m^T(\eta)Q_m r_m(\eta)d\eta + \int_{t-T_2}^t r_s^T(\eta)Q_s r_s(\eta)d\eta \quad (32)$$

$$V_3 = \int_{-T_1}^0 \int_{t+\theta}^t \dot{q}_m^T(\eta)O_m \dot{q}_m(\eta)d\eta d\theta + \int_{-T_2}^0 \int_{t+\theta}^t \dot{q}_s^T(\eta)O_s \dot{q}_s(\eta)d\eta d\theta \quad (33)$$

$$V_4 = \int_{-T_1}^0 \int_{t+\theta}^t \ddot{q}_m^T(\eta)B_m \ddot{q}_m(\eta)d\eta d\theta + \int_{-T_2}^0 \int_{t+\theta}^t \ddot{q}_s^T(\eta)B_s \ddot{q}_s(\eta)d\eta d\theta \quad (34)$$

where $P_m(\omega_1) > 0$, $P_s(\omega_2) > 0$, $Q_m > 0$, $Q_s > 0$, $O_m > 0$, $O_s > 0$, $B_m > 0$, $B_s > 0$. Applying **Lemma 1** and **Lemma 2**, considering **Remark 2** and **3**, assuming at time t , $\varpi_q(t) = \delta$, we can derive follows by using the Makovian infinitesimal operator:

$$\mathcal{L}\dot{V}_1 \leq r^T(A_{11}^T P_\delta + P_\delta^T A_{11} + \sum_{h=1}^n \pi_{\delta h} P_h)r + 2r^T P_\delta(A_{12}\epsilon + A_{13}\dot{q}(t-T) + A_{14}\ddot{q} + A_{15}\dot{q} + A_{16}q + A_{17}) \quad (35)$$

$$\mathcal{L}\dot{V}_2 = r^T Qr - r^T(t-T)H Qr(t-T) \quad (36)$$

$$\mathcal{L}\dot{V}_3 \leq \bar{T}\dot{q}^T O\dot{q} - (q - q(t-T))^T U_1 O(q - q(t-T)) - (q(t-T) - q(t-\bar{T}))^T U_2 O(q(t-T) - q(t-\bar{T})) \quad (37)$$

$$\mathcal{L}\dot{V}_4 \leq \bar{T}\ddot{q}^T B\ddot{q} - (\dot{q} - \dot{q}(t-T))^T U_1 B(\dot{q} - \dot{q}(t-T)) - (\dot{q}(t-T) - \dot{q}(t-\bar{T}))^T U_2 B(\dot{q}(t-T) - \dot{q}(t-\bar{T})) \quad (38)$$

where $P_\delta = \text{diag}[P_m(\omega_2), P_s(\omega_1)]$ at time t , $Q = \text{diag}[Q_m, Q_s]$, $B = \text{diag}[B_m, B_s]$, $O = \text{diag}[O_m, O_s]$, $R = \text{diag}[1 - \bar{d}_1, 1 - \bar{d}_2]$, $U_1 = \text{diag}[\frac{2\bar{T}_1 - \bar{T}_1}{\bar{T}_1^2}, \frac{2\bar{T}_2 - \bar{T}_2}{\bar{T}_2^2}]$, $U_2 = \text{diag}[\frac{\bar{T}_1 + \bar{T}_1}{\bar{T}_1^2}, \frac{\bar{T}_2 + \bar{T}_2}{\bar{T}_2^2}]$. $\bar{T} = \text{diag}[\bar{T}_1, \bar{T}_2]$, $\dot{q}(t-T) - \dot{q}(t-\bar{T}) = \text{diag}[\dot{q}_m(t-T_1) - \dot{q}_m(t-\bar{T}_1), \dot{q}_s(t-T_2) - \dot{q}_s(t-\bar{T}_2)]$, $q(t-T) - q(t-\bar{T}) = \text{diag}[q_m(t-T_1) - q_m(t-\bar{T}_1), q_s(t-T_2) - q_s(t-\bar{T}_2)]$.

It follows (27) that $0 = A_{11}r + A_{12}\epsilon + A_{13}\dot{q}(t-T) + A_{14}\ddot{q} + A_{15}\dot{q} + A_{16}q + A_{17} - \dot{r}$, and set $N = N = [N_1, 0, 0, 0, 0, 0, 0, 0, N_2]^T$. Accordingly,

$$0 = \zeta_1^T N F \zeta_1 + \zeta_1^T N A_{12}\epsilon + \zeta_1^T N A_{13}\dot{q}(t-T) + \zeta_1^T N A_{14}\ddot{q} + \zeta_1^T N A_{15}\dot{q} + \zeta_1^T N A_{16}q + \zeta_1^T N A_{17} \quad (39)$$

where $\zeta_1 = [r, r(t-T), q, q(t-T), q(t-\bar{T}), \dot{q}, \dot{q}(t-T), \dot{q}(t-\bar{T}), \ddot{q}, \ddot{r}]^T$, and $F = [A_{11}, 0, 0, 0, 0, 0, 0, 0, -I]$. Therefore,

$$\begin{aligned} \dot{V} &\leq \mathcal{L}\dot{V}_1 + \mathcal{L}\dot{V}_2 + \mathcal{L}\dot{V}_3 + \mathcal{L}\dot{V}_4 + \zeta_1^T(F^T N^T + NF)\zeta_1 + \\ &2\zeta_1^T N A_{12} \epsilon + 2\zeta_1^T N A_{13} \dot{q}(t-T) + 2\zeta_1^T N A_{14} \ddot{q} + 2\zeta_1^T N A_{15} \dot{q} + \\ &2\zeta_1^T N A_{16} q + 2\zeta_1^T N A_{17} \\ &\leq \mathcal{L}\dot{V}_1 + \mathcal{L}\dot{V}_2 + \mathcal{L}\dot{V}_3 + \mathcal{L}\dot{V}_4 + \zeta_1^T(F^T N^T + NF)\zeta_1 + 2\zeta_1^T N A_{12} \epsilon + \\ &2\zeta_1^T \phi \bar{A}_{17} \zeta_1 + (\bar{A}_{13} + \bar{A}_{14} + \bar{A}_{15} + \bar{A}_{16}) \zeta_1^T N N^T \zeta_1 + \bar{A}_{13} \dot{q}^T(t-T) \dot{q}(t-T) + \\ &\bar{A}_{14} \dot{q}^T \ddot{q} + \bar{A}_{15} \dot{q}^T \dot{q} + \bar{A}_{16} q^T q \\ &= \zeta_1^T \Xi \zeta_1 + 2\zeta_1^T N A_{12} \epsilon \end{aligned} \quad (40)$$

where $\phi = \text{ones}(10, 10)$, $\bar{A}_{13} = \begin{bmatrix} 0 & \bar{d}_2(t)A_m \\ \bar{d}_1(t)A_s & 0 \end{bmatrix}$ and

$$\bar{A}_{14} = \begin{bmatrix} \frac{\bar{M}_m - \bar{M}_m}{2\bar{M}_m} & 0 \\ 0 & \frac{\bar{M}_s - \bar{M}_s}{2\bar{M}_s} \end{bmatrix}, \bar{A}_{15} = \begin{bmatrix} \frac{\bar{C}_m - \bar{C}_m}{2\bar{M}_m} & 0 \\ 0 & \frac{\bar{C}_s - \bar{C}_s}{2\bar{M}_s} \end{bmatrix}, \bar{A}_{16} = \begin{bmatrix} \frac{\bar{D}_m - \bar{D}_m}{2\bar{M}_m} & 0 \\ 0 & \frac{\bar{D}_s - \bar{D}_s}{2\bar{M}_s} \end{bmatrix},$$

$$\bar{A}_{17} = \begin{bmatrix} \frac{\bar{E}_m - \bar{E}_m}{2\bar{M}_m}, \frac{\bar{E}_s - \bar{E}_s}{2\bar{M}_s} \end{bmatrix}^T.$$

$$\Xi = \begin{bmatrix} \Xi_{11} & * & * & * & * & * & * & * & * & * & * \\ \Xi_{21} & \Xi_{22} & * & * & * & * & * & * & * & * & * \\ \Xi_{31} & \Xi_{32} & \Xi_{33} & * & * & * & * & * & * & * & * \\ \Xi_{41} & \Xi_{42} & \Xi_{43} & \Xi_{44} & * & * & * & * & * & * & * \\ \Xi_{51} & \Xi_{52} & \Xi_{53} & \Xi_{54} & \Xi_{55} & * & * & * & * & * & * \\ \Xi_{61} & \Xi_{62} & \Xi_{63} & \Xi_{64} & \Xi_{65} & \Xi_{66} & * & * & * & * & * \\ \Xi_{71} & \Xi_{72} & \Xi_{73} & \Xi_{74} & \Xi_{75} & \Xi_{76} & \Xi_{77} & * & * & * & * \\ \Xi_{81} & \Xi_{82} & \Xi_{83} & \Xi_{84} & \Xi_{85} & \Xi_{86} & \Xi_{87} & \Xi_{88} & * & * & * \\ \Xi_{91} & \Xi_{92} & \Xi_{93} & \Xi_{94} & \Xi_{95} & \Xi_{96} & \Xi_{97} & \Xi_{98} & \Xi_{99} & * & * \\ \Xi_{101} & \Xi_{102} & \Xi_{103} & \Xi_{104} & \Xi_{105} & \Xi_{106} & \Xi_{107} & \Xi_{108} & \Xi_{109} & \Xi_{110} & \Xi_{111} \end{bmatrix}$$

where $\Xi_{11} = P_\delta^T A_{11} + A_{11}^T P_\delta + \sum_{h=1}^n \pi_{\delta h} P_h + Q + N_1^T A_{11}^T + N_1 A_{11} + (\bar{A}_{13} + \bar{A}_{14} + \bar{A}_{15} + \bar{A}_{16}) N_1^T N_1 + 2\bar{A}_{17}$, $\Xi_{21} = 2\bar{A}_{17}$, $\Xi_{31} = P_\delta^T \bar{A}_{16} + 2\bar{A}_{17}$, $\Xi_{41} = \Xi_{51} = \Xi_{81} = 2\bar{A}_{17}$, $\Xi_{61} = P_\delta^T \bar{A}_{15} + 2\bar{A}_{17}$, $\Xi_{71} = P_\delta^T \bar{A}_{13} + 2\bar{A}_{17}$, $\Xi_{91} = P_\delta^T \bar{A}_{14} + 2\bar{A}_{17}$, $\Xi_{101} = N_2^T A_{11}^T - N_1^T + (\bar{A}_{13} + \bar{A}_{14} + \bar{A}_{15} + \bar{A}_{16}) N_1^T N_2 + 2\bar{A}_{17}$, $\Xi_{22} = -HQ + 2\bar{A}_{17}$, $\Xi_{32} = \Xi_{42} = \Xi_{52} = \Xi_{62} = \Xi_{72} = \Xi_{82} = \Xi_{92} = \Xi_{102} = 2\bar{A}_{17}$, $\Xi_{33} = -U_1 O + \bar{A}_{16} + 2\bar{A}_{17}$, $\Xi_{43} = U_1 O + 2\bar{A}_{17}$, $\Xi_{53} = \Xi_{63} = \Xi_{73} = \Xi_{83} = \Xi_{93} = \Xi_{103} = 2\bar{A}_{17}$, $\Xi_{44} = -U_1 O - U_2 O + 2\bar{A}_{17}$, $\Xi_{54} = U_2 O + 2\bar{A}_{17}$, $\Xi_{64} = \Xi_{74} = \Xi_{84} = \Xi_{94} = \Xi_{104} = 2\bar{A}_{17}$, $\Xi_{55} = -U_2 O + 2\bar{A}_{17}$, $\Xi_{65} = \Xi_{75} = \Xi_{85} = \Xi_{95} = \Xi_{105} = 2\bar{A}_{17}$, $\Xi_{66} = \bar{T} O - U_1 B + \bar{A}_{15} + 2\bar{A}_{17}$, $\Xi_{67} = -U_1 B + 2\bar{A}_{17}$, $\Xi_{86} = \Xi_{96} = \Xi_{106} = 2\bar{A}_{17}$, $\Xi_{77} = -U_1 B - U_2 B + \bar{A}_{13} + 2\bar{A}_{17}$, $\Xi_{87} = U_2 B + 2\bar{A}_{17}$, $\Xi_{97} = \Xi_{107} = 2\bar{A}_{17}$, $\Xi_{88} = -U_2 B + 2\bar{A}_{17}$, $\Xi_{98} = \Xi_{108} = 2\bar{A}_{17}$, $\Xi_{99} = \bar{T} B + \bar{A}_{14} + 2\bar{A}_{17}$, $\Xi_{109} = 2\bar{A}_{17}$, $\Xi_{110} = -N_2^T - N_2 + (\bar{A}_{13} + \bar{A}_{14} + \bar{A}_{15} + \bar{A}_{16}) N_2^T N_2 + 2\bar{A}_{17}$.

Adding $e^T e - \gamma^2 \epsilon^T \epsilon - \gamma^2 \dot{q}^T \dot{q}$ to both sides of (40) yields

$$\begin{aligned} \dot{V} + e^T e - \gamma^2 \epsilon^T \epsilon - \gamma^2 \dot{q}^T \dot{q} &\leq \zeta_1^T \Xi \zeta_1 + e^T e - \gamma^2 \epsilon^T \epsilon - \\ \gamma^2 \dot{q}^T \dot{q} + 2r^T P A_{12} \epsilon + 2\zeta_1^T N A_{12} \epsilon &\leq \zeta_2^T \Omega \zeta_2 \end{aligned} \quad (41)$$

where $\zeta_2 = [\zeta_1^T, \epsilon]^T$ and using Schur compliments Ω can be finally transformed to (42)

$$\begin{bmatrix} \Omega_{11} & * & * & * & * & * & * & * & * & * & * & * & * \\ \Xi_{21} & \Xi_{22} & * & * & * & * & * & * & * & * & * & * & * \\ \Xi_{31} & \Xi_{32} & \Xi_{33} & * & * & * & * & * & * & * & * & * & * \\ \Xi_{41} & \Xi_{42} & \Xi_{43} & \Xi_{44} & * & * & * & * & * & * & * & * & * \\ \Xi_{51} & \Xi_{52} & \Xi_{53} & \Xi_{54} & \Xi_{55} & * & * & * & * & * & * & * & * \\ \Xi_{61} & \Xi_{62} & \Xi_{63} & \Xi_{64} & \Xi_{65} & \Xi_{66} & * & * & * & * & * & * & * \\ \Xi_{71} & \Xi_{72} & \Xi_{73} & \Xi_{74} & \Xi_{75} & \Xi_{76} & \Xi_{77} & * & * & * & * & * & * \\ \Xi_{81} & \Xi_{82} & \Xi_{83} & \Xi_{84} & \Xi_{85} & \Xi_{86} & \Xi_{87} & \Xi_{88} & * & * & * & * & * \\ \Xi_{91} & \Xi_{92} & \Xi_{93} & \Xi_{94} & \Xi_{95} & \Xi_{96} & \Xi_{97} & \Xi_{98} & \Omega_{99} & * & * & * & * \\ \Xi_{101} & \Xi_{102} & \Xi_{103} & \Xi_{104} & \Xi_{105} & \Xi_{106} & \Xi_{107} & \Xi_{108} & \Xi_{109} & \Omega_{110} & * & * & * \\ \Omega_{111} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Omega_{1110} & \Omega_{1111} & * & * \\ \Omega_{121} & 0 & \Omega_{123} & 0 & 0 & \Omega_{126} & 0 & 0 & 0 & \Omega_{129} & 0 & \Omega_{1211} & \Omega_{1212} \\ \Omega_{131} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Omega_{1310} & 0 & 0 & \Omega_{1313} \end{bmatrix} < 0 \quad (42)$$

where $\Omega_{11} = P_\delta^T A_{11} + A_{11}^T P_\delta + \sum_{h=1}^n \pi_{\delta h} P_h + Q + N_1^T A_{11}^T + N_1 A_{11} + 2\bar{A}_{17}$, $\Omega_{101} = P_\delta^T \bar{A}_{14} + 2\bar{A}_{17}$, $\Omega_{111} = P_\delta^T A_{12} + N_1 A_{12}$, $\Omega_{121} = X_4$, $\Omega_{131} = N_1$, $\Omega_{123} = X_1$, $\Omega_{126} = X_2$, $\Omega_{129} = X_3$, $\Omega_{99} = \bar{T} B + \bar{A}_{14} + 2\bar{A}_{17} - \gamma^2 I$, $\Omega_{1010} = -N_2^T - N_2 + 2\bar{A}_{17}$, $\Omega_{1110} = N_2 A_{12}$, $\Omega_{1310} = N_2$, $\Omega_{1111} = -\gamma^2 I$, $\Omega_{1211} = X_5$, $\Omega_{1212} = -I$, $\Omega_{1313} = -(\bar{A}_{13} + \bar{A}_{14} + \bar{A}_{15} + \bar{A}_{16}) I$.

Accordingly, we have the following theorem:

Theorem 1: under the time-varying delay condition, the X_1 subsystem is asymptotically stable if there exist matrices $P_\delta > 0$, $O > 0$, $Q > 0$, $R > 0$, $U > 0$ such that LMI (42) holds.

4.2. X_2 subsystem stability

Theorem 2: If there exists positive matrix W and a predefined positive scalar γ such that the following LMI holds

$$\Psi = \begin{bmatrix} \Psi^{11} & * & * & * & * \\ \Psi^{21} & \Psi^{22} & * & * & * \\ 0 & 0 & \Psi^{33} & * & * \\ 0 & 0 & 0 & \Psi^{44} & * \\ \Psi^{51} & 0 & 0 & 0 & \Psi^{55} \end{bmatrix} < 0 \quad (43)$$

where $\Psi^{11} = W A_{21} + I$, $\Psi^{21} = \frac{1}{2} W A_{23}$, $\Psi^{51} = W$, $\Psi^{22} = -\gamma^2 I$, $\Psi^{33} = \left(-\gamma^2 + \frac{1}{2} A_s A_m (1 + \bar{\mu}_1)(1 + \bar{\mu}_2)\right) I$, $\Psi^{44} = \left(-\gamma^2 + \frac{1}{2} (1 + \bar{\mu}_1)(1 + \bar{\mu}_2)\right) I$, $\Psi^{55} = -I$. The position error $e(t)$ in system (28) converges to zero at the steady state.

Proof: Considering Lyapunov function as

$$V^* = e^T W e \quad (44)$$

Thus

$$\dot{V}^* = e^T W (A_{21} e + A_{22} e(t-T) + A_{23} r + A_{24} r(t-T)) \quad (45)$$

Since $\|A_{22}\|^2 = \begin{bmatrix} 0 & (1 - \bar{t}_2) A_s \\ (1 - \bar{t}_1) A_m & 0 \end{bmatrix}^2 \leq A_s A_m (1 + \bar{\mu}_1)(1 + \bar{\mu}_2)$

and $\|A_{24}\|^2 = \begin{bmatrix} 0 & -(1 - \bar{t}_2) I \\ -(1 - \bar{t}_1) I & 0 \end{bmatrix}^2 \leq (1 + \bar{\mu}_1)(1 + \bar{\mu}_2) I$

Therefore

$$e^T W A_{22} e(t-T) \leq \frac{1}{2} \|W e\|^2 + \frac{1}{2} A_s A_m (1 + \bar{\mu}_1)(1 + \bar{\mu}_2) \|e(t-T)\|^2 \quad (46)$$

$$e^T W A_{14} r(t-T) \leq \frac{1}{2} \|W e\|^2 + \frac{1}{2} (1 + \bar{\mu}_1)(1 + \bar{\mu}_2) \|r(t-T)\|^2 \quad (47)$$

Consider the following equation

$$\begin{aligned} \int_0^t (e^T(\eta) e(\eta) - \gamma^2 \zeta^T(\eta) \zeta(\eta)) d\eta &= \int_0^t (e^T(\eta) e(\eta) - \\ \gamma^2 \zeta^T(\eta) \zeta(\eta) + \dot{V}^*) d\eta - V^* &\leq \int_0^t \mathcal{E}^T \Psi \mathcal{E} d\eta - \frac{1}{2} e^T W e \end{aligned} \quad (48)$$

where $\mathcal{E} = [e(t), r(t), e(t - T_{1,2}(t)), r(t - T_{1,2}(t))]^T$. Based on (43), we have $\int_0^t (e^T(\eta) e(\eta) - \gamma^2 \zeta^T(\eta) \zeta(\eta)) d\eta \leq 0$. Therefore, $\int_0^t e^T(\eta) e(\eta) d\eta \leq \int_0^t \gamma^2 \zeta^T(\eta) \zeta(\eta) d\eta < \infty$. Moreover, $e(t) \in L_2$, which determines $e(t) \rightarrow 0$ when $t \rightarrow \infty$.

V. EXPERIMENTAL RESULTS

In this section, a series of experimental results are provided. The process of the proposed control algorithm is shown in Fig. 2, and the applied experimental platform consists of two 3-DOF haptic devices as shown in Fig. 3.

In the experiments, the proposed Type-2 T-S fuzzy modeling approach is used to transforming the nonlinear system with unknown dynamics and uncertainties to a combination of multiple linear system while the unmeasurable disturbance such as gravity can be fully compensated. Accordingly, to enlarge the influence the unknown disturbance, especially the gravity, a heavy iron load (around 0.2 kg) is hanged at the end effector of the slave to indicate a heavy manipulator of slave robot with unknown gravity dynamics. The experiments are performed in the presence of different scenarios. The time delays are 300 ms with 100 ms variation.

Fig. 2. Process of the overall control algorithm

Fig. 3. Experimental platform

For a Master or a Slave of the teleoperation system with the degree of freedom $n^f = 3$, a Type-2 T-S fuzzy model with $c = 9$ fuzzy rules is constructed from 19366 data samples where the sampling period is $\Delta T = 0.001$. We present the coefficients of the first fuzzy rules of Master and Slave as an example:

For Master: $x_{m,c}^1 = [-0.0003 \quad -0.0005 \quad -0.0001 \quad 0.0905 \quad -0.1084 \quad 0.0353 \quad 0.3970 \quad 0.9805 \quad 0.8634]^T, \Delta\mu_m^1 = 0.05$
 $\underline{M}_m^1 = \text{diag}[1.1720, 0.5231, 1.3421]$, $\overline{M}_m^1 = \text{diag}[1.2201, 0.6103, 1.4700]$, $\underline{C}_m^1 = \text{diag}[-0.5170, -0.6215, -0.3175]$, $\overline{C}_m^1 = \text{diag}[-0.4931, -0.5874, -0.1862]$, $\underline{D}_m^1 = \text{diag}[0.1332,$

$0.0501, -0.1366]$, $\overline{D}_m^1 = \text{diag}[0.2741, 0.0681, -0.1250]$, $\underline{E}_m^1 = [-0.1745 \quad -0.0500 \quad 0.0135]^T$, $\overline{E}_m^1 = [0.0288 \quad 0.1524 \quad 0.2183]^T$
 For Slave: $x_{s,c}^1 = [0.0012 \times 10^{-3} \quad -0.0053 \times 10^{-3} \quad -0.0214 \times 10^{-3} \quad 0.0279 \quad -0.0035 \quad -0.0034 \quad -0.2104 \quad 0.7262 \quad 0.6444]^T, \Delta\mu_s^1 = 0.05,$

$\underline{M}_s^1 = \text{diag}[1.6214, 0.5935, 1.5721]$, $\overline{M}_s^1 = \text{diag}[1.7037, 0.7021, 1.6230]$, $\underline{C}_s^1 = \text{diag}[0.1428, 0.2917, -0.1145]$, $\overline{C}_s^1 = \text{diag}[0.1430, 0.2979, -0.1129]$, $\underline{D}_s^1 = \text{diag}[0.0029, 0.3501, 0.1792]$, $\overline{D}_s^1 = \text{diag}[0.0192, 0.4471, 0.1847]$, $\underline{E}_s^1 = [-0.0936, -0.0083, -0.0731]^T$, $\overline{E}_s^1 = [0.1064, 0.1897, 0.1314]^T$

The curves of errors and the value of root mean square errors (RMSEs) of the constructed Type-2 fuzzy models for Master and Slave are shown in Fig. 4.

Fig. 4. Errors of Type-2 T-S fuzzy models for Master and Slave

Based on the derived values of M_i^l , C_i^l , D_i^l and E_i^l ($l = 1, 2, \dots, 9$) from the designed fuzzy rules, the main control gains of the designed control laws, $\overline{k}_1^l$, $\underline{k}_1^l$, $\overline{k}_2^l$, $\underline{k}_2^l$, $\overline{k}_m^l$, $\underline{k}_m^l$, and $\overline{k}_s^l$, $\underline{k}_s^l$ can be derived using the LMI toolbox. The fuzzy gains are derived as

$\overline{k}_1^{1-9} : \text{diag}[1.8572, 1.8757, 1.7874]$, $\text{diag}[1.8994, 1.9011, 1.8413]$,
 $\text{diag}[1.9032, 1.8963, 1.9207]$, $\text{diag}[1.8657, 1.8496, 1.8373]$,
 $\text{diag}[1.8555, 1.8212, 1.8064]$, $\text{diag}[1.8992, 1.9181, 1.8747]$,
 $\text{diag}[1.8015, 1.8031, 1.7669]$, $\text{diag}[1.9145, 1.9742, 1.8907]$,
 $\text{diag}[2.1042, 2.0102, 1.9931]$.
 $\underline{k}_1^{1-9} : \text{diag}[1.8835, 1.8993, 1.8062]$, $\text{diag}[1.8427, 1.8591, 1.7636]$,
 $\text{diag}[1.8429, 1.8165, 1.8531]$, $\text{diag}[1.8555, 1.8212, 1.8064]$,
 $\text{diag}[1.8743, 1.9068, 1.8338]$, $\text{diag}[1.8420, 1.8267, 1.7778]$,
 $\text{diag}[1.7135, 1.7132, 1.6999]$, $\text{diag}[1.7849, 1.7244, 1.8213]$,
 $\text{diag}[1.8942, 1.9209, 1.7138]$.
 $\overline{k}_2^{1-9} : \text{diag}[1.3201, 1.4231, 1.4463]$, $\text{diag}[1.6651, 1.7136, 1.6331]$,
 $\text{diag}[1.4649, 1.6335, 1.1597]$, $\text{diag}[1.4930, 1.1449, 1.5100]$,
 $\text{diag}[1.7130, 1.7377, 1.6162]$, $\text{diag}[1.3789, 1.6222, 1.0789]$,
 $\text{diag}[2.0030, 1.9136, 1.9269]$, $\text{diag}[1.6743, 1.3814, 1.4583]$,
 $\text{diag}[1.2947, 1.5553, 1.6413]$.
 $\underline{k}_2^{1-9} : \text{diag}[1.5861, 1.4965, 1.3349]$, $\text{diag}[1.3315, 1.5594, 1.4592]$,
 $\text{diag}[0.8556, 1.2564, 1.3125]$, $\text{diag}[1.2307, 1.0155, 1.4065]$,
 $\text{diag}[1.6175, 1.3726, 1.3048]$, $\text{diag}[1.2334, 1.5826, 0.9543]$,
 $\text{diag}[1.5387, 1.6235, 1.7891]$, $\text{diag}[1.5921, 1.2101, 1.2709]$,
 $\text{diag}[0.9996, 1.2719, 1.5328]$.
 $\overline{k}_m^{1-9} : \text{diag}[0.7249, 0.6360, 1.0597]$, $\text{diag}[0.8250, 0.8010, 1.1369]$,
 $\text{diag}[0.8066, 0.9445, 0.8029]$, $\text{diag}[0.7831, 0.9075, 0.9511]$,
 $\text{diag}[0.6218, 0.4907, 0.8875]$, $\text{diag}[1.0047, 0.9947, 1.1897]$,
 $\text{diag}[0.8024, 0.9172, 0.8438]$, $\text{diag}[0.7034, 0.8421, 0.7490]$,
 $\text{diag}[0.6479, 0.9821, 0.7082]$.
 $\underline{k}_m^{1-9} : \text{diag}[0.6082, 0.5187, 0.9945]$, $\text{diag}[0.5329, 0.6196, 0.8936]$,
 $\text{diag}[0.5563, 0.3164, 0.4093]$, $\text{diag}[0.6476, 0.8405, 0.8265]$,
 $\text{diag}[0.5351, 0.3965, 0.6241]$, $\text{diag}[0.8646, 0.8978, 1.1354]$.

$\text{diag}[0.6325, 0.6732, 0.7992]$, $\text{diag}[0.5320, 0.6142, 0.4912]$,
 $\text{diag}[0.6394, 0.5242, 0.5201]$.
 $\bar{k}_s^{-1-9}$: $\text{diag}[0.9411, 0.8053, 0.7744]$, $\text{diag}[0.9947, 0.4013, 0.7692]$,
 $\text{diag}[1.5883, 1.0508, 1.1810]$, $\text{diag}[1.0756, 1.1990, 0.8378]$,
 $\text{diag}[0.5399, 0.8990, 0.9955]$, $\text{diag}[1.2750, 0.5310, 1.2775]$,
 $\text{diag}[1.1333, 0.9040, 0.9201]$, $\text{diag}[0.7854, 0.7922, 0.7998]$,
 $\text{diag}[0.7479, 0.6929, 0.8001]$.
 $\bar{k}_s^{-1-9}$: $\text{diag}[0.5815, 0.7076, 0.9310]$, $\text{diag}[0.4730, 0.5087, 0.5177]$,
 $\text{diag}[0.7668, 0.5378, 0.9906]$, $\text{diag}[0.7165, 0.9603, 0.6944]$,
 $\text{diag}[0.4032, 0.3656, 0.5433]$, $\text{diag}[1.0780, 0.4453, 0.7771]$,
 $\text{diag}[0.6396, 0.6038, 0.5093]$, $\text{diag}[0.5127, 0.4941, 0.7210]$,
 $\text{diag}[0.6014, 0.5946, 0.6538]$.

In addition, Υ in (42) is $2.6730\text{e-}10$ that is small enough for the overall system stability. For the other parameters, $\Lambda_m = \Lambda_s = 0.4$. $\rho_{m0} = \rho_{s0} = 2$, $\rho_{m\infty} = \rho_{s\infty} = 0.5$, $p = 0.99$.

5.1 Free motion

In this subsection, our target is to let the slave with a heavy manipulator perform fine free motion just using the proposed control laws for linear system. Moreover, the operator is required neither to feel large force feedback nor to apply large force to drive the heavy slave to move to the designated spot. If so, it means the proposed Type-2 T-S fuzzy model based algorithm can fully estimate and eliminate the unknown dynamic uncertainties and disturbances. To better demonstrate the superiority of the proposed approach, the designed system is compared to another two systems with novel approaches, a four-channel T-S fuzzy modeled system in [21] (System A) and an adaptive system based on an auxiliary switched filter in [25] (System B). The parameter configuration of the above two systems basically follows the recommendation of references [21] and [25].

Fig. 5. System A (free motion)

Fig. 5 shows the performance of System A. System A uses T-S fuzzy model to transform a nonlinear system to the combination of multiple linear systems without considering any dynamic uncertainties and assumes the exact gravity model to be applied to the control laws. However, under the condition of Fig.3, the exact gravity model is impossible to derive. Therefore, the position tracking errors are largely enlarged by the hanged heavy load. Especially for the joint 2, the slave joint can be hardly moved by the master command signals and is unable to follow the master to move to the designated spot. Furthermore, the operator must apply large torques (Maximum 1.5 Nm) to drive the master robot to the designated spot. The heavy load renders the whole system very redundant. The Root Mean Square Errors (RMSEs) of position and torque tracking errors (Master – Slave) in Fig. 5 are list in Table 1.

Table 1. RMSE (tracking errors for Fig. 5)

RMSE	Position	Torque
Joint 1	6.9581	25.8577
Joint 2	82.4937	7.9405
Joint 3	51.8073	2.2712

Fig. 6 shows the performance of System B. the new auxiliary switched filter applied in System B has its superiority that can force the position errors to converge to zero. Therefore, even without gravity models and affected by the heavy load, the system can still have good position tracking performance. Especially for the joint 1 which is less affected by the heavy load, the position tracking errors are nearly zero. Only the joint 2 and joint 3 has obvious tracking errors but also reasonable. However, the adaptive control algorithm designed for dynamic uncertainties in System B is velocity dampers with variable gains. Surely the dampers can guarantee the system stability and the variable gains can enhance transparency to some extent. However, by assuming or setting plenty of upper bounds for the dynamic uncertainties, this algorithm is too conservative for dynamic uncertainties compensation. When remotely controlling the slave with a heavy end effector to conduct free motion, due to the increased system mass and gravity, the operator still feels large force feedback and must apply large torques (More than 1 Nm) to drive the slave to reach the designated spot.

Fig. 6. System B (free motion)

Table 2. RMSE (tracking errors for Fig. 6)

RMSE	Position	Torque
Joint 1	1.8939	5.2730
Joint 2	5.3794	5.0391
Joint 3	3.9912	3.9087

Fig. 7. The proposed system (free motion)

Table 3. RMSE (tracking errors for Fig. 7)

RMSE	Position	Torque
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Joint 1	1.2914	0.1125
Joint 2	1.7022	0.1309
Joint 3	1.7129	0.1213

Fig. 7 shows the performance of the proposed Type-2 fuzzy model based system. The most apparent difference from the above two systems is that the operator does not feel large force feedback when driving the master to the extent that the operator does not need to apply large forces to drive the overall system. For the joint 2, the main joint affected by gravity, the maximum applied torque is less than 0.1 Nm that is much smaller than that of System A and B. Moreover, accurate position synchronization is achieved in the proposed system with the tracking errors neighboring zero. Notice that the proposed control law does not apply any advanced control algorithms such as the switched filter in System B but still derive a better position tracking. It means the applied the Type-2 fuzzy model based control algorithm has superiority on modelling the overall nonlinear system, compensating for dynamic uncertainties, and enable the control laws for linear systems to derive high work performance in nonlinear systems.

5.2 Hard contact

In this subsection, the slave with the heavy load is controlled to conduct free motion at first and then it contacts to a solid wall. A transparent system with a heavy manipulator is required to provide the operator little force feedback at the free motion stage but to provide the operator a real force feedback when contacting the solid wall so that the operator can accurately determine whether the slave's motion is impeded by the environmental object and then the human force can track the environmental force. The proposed system is compared with a four-channel wave-based teleoperation system that uses Mamdani fuzzy model to estimate the system uncertainties in [28] (System C). Fig. 8 shows the performance of System C (0s-9s free motion, 9s-18s hard contact). As introduced before, the traditional Mamdani model provides degraded performance and has no superiority on dealing with system uncertainties over Type-2 Fuzzy model. Therefore, both in the free motion and in the hard contact, the operator keeps feeling large force feedback so that without vision feedback, the operator has no idea on whether the slave is contacting an environmental object or not. Moreover, the position tracking is also influenced by the large dynamic uncertainties. Fig. 8 illustrates that the four-channel waved-based method provided in System C has superiority on sharply-varying delays but has no priority on dynamic uncertainties.

Fig. 8. System C (constrained motion)

Table 4. RMSE (tracking errors for Fig. 8)

RMSE	Position	Torque
Joint 1	3.6592	1.3245
Joint 2	9.8290	1.9077
Joint 3	7.2197	1.4037

Fig. 9 shows the performance of the proposed system (0s-10s free motion, 10s-18s hard contact). The figure clearly shows that during the free motion the operator basically receives no force feedback. When contacting to the solid wall, the applied torque control method immediately allows the operator to feel the remote feedback and good position tracking is also achieved without large errors even during the hard contact. The experiment clearly shows that the proposed method owns high ability on separating the dynamic disturbances and the necessary environmental force.

Fig. 9. Proposed system (constrained motion)

Table 5. RMSE (tracking errors for Fig. 9)

RMSE	Position	Torque
Joint 1	0.2971	0.1102
Joint 2	0.2310	0.1089
Joint 3	0.3021	0.1016

5.3. Combined with wave-based TDPA (Sharply-varying delays)

Under the extreme case that large and sharply varying delays occur, the proposed system can directly combine with the wave-based TDPA in [26]-[27] to guarantee the position and torque tracking. Fig. 10 shows the system performance (0s-14s free motion, 14s-18s hard contact) under sharply-varying delays (1500 ms with 1000 ms variation). The system firstly conduct a free motion and then contact to the solid wall. Due to the large delays, the performance inevitably degraded but still reasonable, and the system remains stable under such large time delays. Especially, during free motion, the position tracking is still highly accurate.

Fig. 10. Combined with wave-based TDPA (sharply varying delays)

Table 6. RMSE (tracking errors for Fig. 10)

RMSE	Position	Torque
Joint 1	6.9781	2.5235
Joint 2	6.3275	2.0005
Joint 3	7.8803	2.9471

VI. CONCLUSION

In this paper, a new Type-2 T-S fuzzy logic based framework is proposed for bilateral teleoperation system under the influences of large uncertainties and time-varying delays. Interval Type-2 T-S fuzzy models are constructed that transforms the nonlinear teleoperation system to a combination of multiple linear teleoperation systems to facilitate controller designs. Compared with previous work, the proposed Type-2 fuzzy model based approach has superiority on accurate compensation for the dynamic uncertainties and disturbances. In this way, the control laws designed for linear systems can be applied. The torque tracking method based on the prescribed performance control is applied to improve the operator's force perception for the environment when large dynamic uncertainties exist. Moreover, interval time-varying delays in bilateral teleoperation is analysed and the control gains are no longer prescribed in particular form, but expressed as LMIs. By using the LMI toolbox, the optimized parameters can be efficiently calculated. The experiments are performed on two haptic devices and experimental comparisons show the effectiveness of the proposed strategies.

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