# EYE-TRACKING AS A TOOL FOR INVESTIGATING MATHEMATICAL CREATIVITY.

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*Mathematical creativity as a key ability in our increasingly automated and interconnected, high-technology based society and economy is increasingly in the focus of mathematics education research. The recent scientific discussion in this domain is shifting from a product view, on written solutions and drawings, to a process view, which aims to investigate the different stages of how students come up with creative ideas. The latter is, however, a challenge. In this theoretical-methodological paper, we present and discuss the opportunities that eye-tracking offers for studying creativity in a process view. We discuss in which way eye-tracking allows to obtain novel answers to the questions of how original ideas come up, how they evolve and what leads to the so-called Eureka!-moment. We focus on video-based eye tracking approaches, discuss pros and cons of screen-based and mobile eye tracking, and illustrate methods of data analysis and their benefits for research on mathematical creativity.*

*Key words: Mathematical Creativity, Research Methods, Eye-Tracking, Eye Movements, MSTs, geometry, proof*

## INTRODUCTION

Creativity is an ability which is significant for generating novelties, finding original ideas, or treading new paths of thinking. This is crucially important not only for mathematics but for all STEM areas (science, technology, engineering, and mathematics) in the increasingly automated and interconnected high-technology based societies and economies that we are and will be living in in the next decades. We see a trend in mathematics education research increasingly focusing on creativity: It is no longer sufficient for students to solve problems with routine schemes or familiar heurisms. Educators want to enable students to think "out of the box", to connect different topics whilst solving problems, to have "aha!" experiences.

Thus, research has focused on how to foster and—as a prerequisite—how to study mathematical creativity. Researchers have started investigating and discussing methods to study mathematical creativity (e.g., Joklitschke, Rott & Schindler, 2016; Schindler et al., 2016). Research has mainly focused on students' products—their written solutions and drawings (e.g., Kattou et al., 2013; Leikin & Lev, 2013). However, the recent scientific discussion has lifted that a profound analysis of mathematical creativity should furthermore consider students' *processes* while working on problems (e.g., Joklitschke et al., 2016) and answer the question of how students solve problems creatively. This will allow for deeper insights on how creative ideas emerge, what triggers them, and how students can be supported in solving problems creatively.

Investigating mathematical creativity in a process view is, however, a challenge for researchers. The existing scientific methods (e.g., thinking-aloud interviews, or videotaping students' problem-solving) each have their weaknesses. They often miss certain parts of the thinking processes leading to the question of which methods can complement the existing ones. We found that eye-tracking offers various opportunities to investigate mathematical creativity in a process view. Eye-tracking is a method increasingly gaining popularity in educational research (Andrá et al., 2015) and is accessible more than ever before (Holmkvist et al., 2011). Eye-tracking goggles or remote devices mounted to a computer screen "track" the movements of a person's eyes with cameras. Eye-tracking technology enables researchers to see where exactly persons look at while, for instance, solving a mathematical problem. Eye-tracking offers various opportunities for researchers, as Holmkvist et al. state, "There is no doubt that it is useful to record eye-movements, it advances science and leads to technological innovations" (p. 1).

## MATHEMATICAL CREATIVITY

Today's view on mathematical creativity is predominantly influenced by Guilford's theory of intelligence (Guilford, 1967), which sees creativity as one dimension of intelligence. Guilford's theory emphasizes divergent thinking, the ability to find unique and manifold ideas, which he pictures as "most relevant to creative performance" (p. 169). Divergent thinking in this approach is conceptualized and evaluated in four dimensions: *fluency*, as the number of solutions; *flexibility*, as the diversity of produced solutions; *originality*, as the uniqueness of produced solutions; and *elaboration*, as the level of detail.

Guilford's approach has largely been used in educational research and mathematics education research in particular (e.g., Kattou et al., 2013; Leikin & Lev, 2013). Here, researchers draw on so-called Multiple Solution Tasks (MSTs); mathematical problems that can be solved in different ways. The participants working on MSTs are asked to solve them in as many ways as possible. MSTs can be used both for evaluating and fostering mathematical creativity among students. Tests to evaluate mathematical creativity mostly draw on MSTs and refer to Guilford's categories of fluency, flexibility, and originality for scoring creativity; considering correct and complete solutions.

Recently, further ideas on mathematical creativity and its evaluation have emerged in the scientific discussion in mathematics education (e.g., Schindler et al., 2016; Liljedahl, 2013). Whereas most previous research focused on creativity as a product or on creativity as an attribute of a person, the idea to treat and evaluate creativity as a *process* (Rhodes, 1961) has increasingly attracted attention. Research with a process perspective on creativity mostly draws on Poincaré's (1948) and Hadamard's (1945) ideas, focusing on the processes of preparation, incubation, illumination, and verification (Sriraman, 2009). Contemporary research in mathematics education on creativity asks the question of how such illumination emerges and "what is the nature of this phenomenon?" (Liljedahl, 2013, p. 253). We see that questions such as how original ideas come up, or what leads to the so-called Eureka!-moment should be further investigated. Furthermore, we see think that fluency and flexibility—characteristics of creativity usually considered in a product perspective—should be reconceptualized in a process view: Questions such as what it means to flexibly work on a problem, or how students can be fostered in thinking flexibly when working on a MST, are worth investigating. Eye-tracking offers the possibility to get groundbreaking answers to these questions and new insights that mathematics education research has not gained so far (Schindler et al., 2016).

## EYE-TRACKING

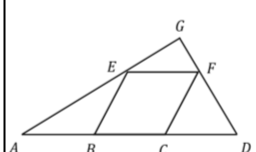
Eye-tracking is a technique to capture persons' eye movements when they are looking at stimuli at hand (Chen, 2011). It can capture—among others—participants' fixations and saccades, which are typically analyzed in eye-tracking research (Salvucci & Goldberg, 2012). *Fixations* are moments when the eye remains relatively still and focuses—consciously or not—stably on a certain focus point or a small area. *Saccades* are fast eye-movements in between fixations (Chen, 2011). Humans make—intentional or triggered by a reflex—about three of such saccades per second on average (Jang et al., 2014).

Research using eye-tracking technology has increasingly gained popularity over the last decade (Andrá et al., 2015; Salvucci & Goldberg, 2012). It draws on the so-called "eye-mind" hypothesis (Just & Carpenter, 1976) meaning that what a person looks at is in the focus of the person's cognitive processes and that a person's eye-movements are tightly related to their cognitive processes (Jang et al., 2014). Accordingly, eye-movements are understood to offer a "dynamic trace" of a person's attention and its shifts (p. 318) as well as information on persons' interests and intentions (Zhai et al., 1999). In particular, fixations indicate attention and cognitive processing of information; saccades hint at shifts of attention; together, they indicate how persons acquire information and what information they are attending to (Andrá et al., 2015; Jang et al., 2014; Chen, 2011). They even hint at persons' *intentions* and may predict their *future actions*: If a person, for instance, holds her hands under the water faucet and then fixates the soap, this presumably reflects the intention to use the soap and may—not necessarily—be followed by an action (the person taking and using the soap). However, the person can also drop this intention, e.g., when the person realizes that the soap dish is empty, and not carry out the action. In our empirical eye-tracking research, we found similar effects: When the participants, e.g., pointed at a particular point in the figure of a geometry MST, the fixation onto this point and, thus, their visual attention preceded the action. The fixations did not only reflect students' attention but also indicated their intention for further actions.

## EYE-TRACKING IN CREATIVITY RESEARCH

Relatively recently, eye-tracking has been applied for investigating mathematical creativity (Schindler et al., 2016; Muldner & Burleston, 2015)—with different techniques and intentions. In common, researchers have the aim to improve existing methods studying mathematical creativity. In both studies, eye-tracking was used in a setting where the participants worked on MSTs dealing with proof in geometry (as shown in Fig. 1).

Solve the following problem. Can you find different ways to solve the problem? Find as many ways as possible.

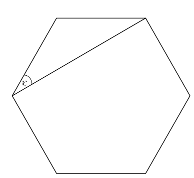



In triangle AGD, points E and F are on AG and DG respectively, and points B and C are on AD.

Given that EF = FC = CB = BE and AB = BC = CD, prove that triangle AGD is a right triangle.

**Triangle MST** (e.g., Muldner & Burleston, 2015)

Solve the following problem. Can you find different ways to solve the problem? Find as many ways as possible.



This figure is an equilateral hexagon. How big is the angle ε?

Remember, in an equilateral hexagon, all sides have the same length and all angles have the same size, which is 120°.

**Hexagon MST** (e.g., Schindler et al., 2016)

Figure 1: Multiple Solution Tasks (MSTs) that have been applied in eye-tracking studies.

In our view, eye-tracking holds huge potential and is especially beneficial for investigating mathematical creativity using geometry proof MSTs. Much more than in, e.g., arithmetic MSTs it is likely in geometry MSTs that the students' cognitive attention lies on the same focus that they are visually paying attention to. Eye-tracking can, amongst others, contribute to getting closer to students' thinking processes while solving geometry MSTs, it can give insights into the emergence of new lines of thought and ideas, it can hint at what characterizes flexibility in the creative process, and help researchers to find patterns of creative problem-solving processes.

## Technical implementation of eye-tracking in creativity research

Eye-tracking approaches determine the orientation of the eye ball and can be classified into three categories: contact lens, electro-oculogram, and video-based (Lupu & Ungureanu, 2013). The first two categories are strongly invasive—participants need to wear contact lenses with mirrors, or electrodes near the eye. In video based approaches, the pupil is located in a video stream. For research on mathematical creativity, video based approaches are expedient since they are a (relatively) non-intrusive method. Most commonly *static eye-trackers* or *head-mounted eye-trackers* are used (cf. Holmkvist et al., 2011).

Among *static eye-trackers*, *screen-based eye-tracking* (as used by Muldner & Burleston, 2015) has recently gained popularity, where a device is attached to a screen and views the participant's eye from the distance (Holmkvist et al., 2011). This offers the advantages of unobtrusiveness and lower demands on the data processing (compared to mobile eye-tracking), in particular since it is straightforward to relate gaze directions to the displayed content on a screen. Additionally, screen-based eye-tracking is usually much more affordable than mobile eye-tracking. If fully fledged hard- and software are bought, the screen-based technology is many times cheaper than the mobile one at the current state.

Among *head-mounted eye-trackers*, *mobile eye-tracking with goggles* (as used by Schindler et al., 2016), which are relatively lightweight (approx. 100 g) is increasingly used in educational research. It allows students to work on paper and pencil tasks. Especially in geometry MSTs, it may be unfamiliar for the students to solve MSTs on a computer screen, where scribblings, writings, and drawings have to be undertaken differently from what many students are used to. In mobile eye-tracking, students can work on the sheet of paper, move it around, can move their heads and body relatively freely. We see that mobile eye-tracking avoids the influence that screen-based eye-tracking may have on students' creative processes, while is provides accuracy and precision similar to remote eye-trackers.

## Data analysis of eye-tracking in creativity research

The analysis of eye-tracking data provides a view on internal cognitive processes underlying students' actions, which is not available from a study of products or the observation of students. We illustrate three methods of eye-tracking analysis that are useful for studying creativity. They apply to both screen-based and mobile eye-tracking.

*Gaze overlaid videos.* This qualitative analysis draws on gaze overlaid, augmented videos, where participants' focus is visualized by a circle wandering in the way their eye movement does. Researchers can have a detailed look at what students are paying attention to and how their focus of attention shifts. Schindler et al. (2016) found that the analysis of eye-tracking overlaid videos can contribute to revealing "how new, creative ideas evolved, to reconstructing approaches that were complex and whose written/drawn descriptions did not allow to clearly reconstruct them, and to evaluating the degree of elaboration of students' approaches" (p. 168). An issue of analyzing gaze overlaid videos is that the analysis is very time consuming. Other methods of analysis can be taken into account, e.g., attention maps, in order to preselect relevant episodes or approaches.

*Attention Maps (Heat Maps) and Scan Paths.* Attention maps are a representation of eye-tracking data most often visualized as *heat maps*. Heat maps show the distribution of gaze points over a certain period, projected onto the task sheet. Areas where the participant looked at often are usually colored in warm colors (red), whereas areas where only few fixations applied, are marked in colder colors (blue) (cf. Holmkvist et al., 2011). Heat maps do not preserve sequential information from the eye-tracking device. In creativity research, heat maps can be used to get an overview on what kind of approach a student used when solving an MST: It provides researchers with an image of what the main foci of attention were. This can be used to categorize students' approaches or to sort them—for instance, for preselecting approaches for a further qualitative analysis. Similar to attention maps, *scan paths* provide information on where the participants looked at. In addition, they display the sequence of fixations by numbered, connected points (representing the fixations) that are connected through lines or arrows in the order of appearance. It is even possible to represent the fixation times through the size of the displayed points. Scan paths can be used to portray students' approaches. They even allow for a glimpse on the problem-solving *process* and, thus, for studying creativity in a process view.

*Statistical measures.* Data from eye-tracking devices can furthermore be quantified, which can be understood as the counterpart the qualitative analyses mentioned above. The analysis is conducted based on the data, not on the visualization. What can be measured is, e.g., the movement directions and their velocity, the position duration, the numbers of saccades or fixations, and the saccadic latency. Muldner and Burleston (2015) conducted statistical analyses of measures when comparing a low- and high-creativity group of participants. They, e.g., found that low creativity students had a significantly shorter saccade path length indicating that these participants "may have focused more locally when generating their proofs, since shorter saccades suggest they were looking at (and fixating on) objects in close proximity" (p. 135). Here, the benefits for research on mathematical creativity become apparent: It enables researchers to, e.g., find quantifiable differences in the eye movement data of different groups of participants solving MSTs.

## OUTLOOK

This paper only gave a glimpse on the opportunities that eye-tracking holds for investigating mathematical creativity. We think that empirical research must prove if our ideas and visions on how to use eye-tracking for investigating creativity can hold or have to be revised; and how they can be specified. Furthermore, the technology offers more interesting and valuable ways of analysis that we were—due to space limitations—not able to elaborate on. Finally, we see that automatic analysis through pattern recognition and machine learning approaches offers various opportunities to analyze students' creativity both in a product and in a process view in the future. This can contribute to effectively and quickly analyze students' problem-solving, to finally foster them adequately in their creativity, and accordingly prepare them for their future lives in modern, creative societies.

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