THE NATURE AND USE OF THEORIES IN STATISTICS EDUCATION – LOOKING BACK, LOOKING FORWARD

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Theories have a significant role for scientific work—also for statistics education research (SER). This paper elaborates on the use of theories in SER, based on findings of a literature review on the nature and use of theories in SER. In particular, we address theoretical issues and possible directions to further theory development in SER. Subsequently, we discuss five themes that in our view need further attention in SER.

INTRODUCTION

Theories are significant for scientific work; theories emerge from scientific work and they guide scientific work. Theories help researchers to understand and explain complex phenomena and to predict behavior to a certain extent (Bikner-Ahsbahs & Prediger, 2010; Lester, 2010). The importance of theories for the development of a scientific research discourse was the starting point for a literature review, in which we focused on mapping and describing types of theories used in SER (Nilsson, Schindler, & Bakker, 2017). In this paper, we draw on the outcomes of this literature review. We describe different types of theories found to be used in SER and elaborate on possible future directions of theory development in SER.

THEORIES AS USED IN STATISTICS EDUCATION RESEARCH

In Nilsson et al. (2017), we categorize theories being used in empirical studies on students' learning of statistics or probability at the primary and secondary school level. Our review includes publications from 2004 until February 2015 from four journals in mathematics and statistics education research (Statistics Education Research Journal, Educational Studies of Mathematics, Journal for Research in Mathematics Education, and Mathematical Thinking and Learning). Our review includes articles about empirical studies, which focus on school students (primary and secondary level) and which were written in English. The review came to include 35 articles. Drawing on this review, we describe types of theories used in SER.

Theories of statistics (ToS)

There are theories in SER, which contain some *domain-specific* theoretical discussion on statistical subject matter. Digging deeper in the category of ToS, we discerned two sub-categories: *Statistical Product Theories* (SPdT) and *Statistical Process Theories* (SPcT). SPdT are theories that address the products of statistics. They concern a single or a limited set of the big ideas of statistics and/or probability, such as variability, average, samples, and graphs (Shaughnessy, 2007), randomness and independence (Gal, 2005), or the role of sample space and comparing probabilities (Nilsson, 2009). Such theories are used in approx. three quarters of the articles in the review. In contrast to SPdT, SPcT cover theoretical constructs that focus on steps and processes involved in statistical investigations such as students becoming engaged in formulating statistical and probabilistic questions, collecting data, analyzing data, and drawing data-based conclusions and inferences (Paparistodemou & Meletiou-Mavrotheris, 2008). Approx. half of the articles in the literature review referred to such theories

Comparing studies in SPdT with studies in SPcT, we note that the majority of studies focusing on statistical products conceptualize these constructs from a student perspective. The focus is on students' conceptions and, especially, on detecting shortcomings and misconceptions. On the contrary, the theoretical underpinning of statistical processes is mainly done by describing processes of statistical work, which is based on the practice developed within the discipline (e.g., Wild & Pfannkuch, 1999).

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Theories with a Didactical Focus (TDF)

This category comprises theoretical approaches in which didactical aspects are taken into consideration as means to support learning. The theories may concern a specific design principle in the field of statistics education, for example the idea of *growing samples* (Ben-Zvi, 2012), or approaches related to computer-based learning, inquiry-based learning, problem-based teaching, or Realistic Mathematics Education (RME). These theories typically have a prescriptive nature and address the design of learning processes and learning environments. Other theories in this category are used to understand *language* and its influence in statistics learning, *contexts* and their influence on students' understanding and their learning process, *technology use* in the mathematics learning process, and—in a broader sense—*representations* and their role in the learning of statistics. All in all, slightly more than half of all articles reviewed made reference to TDF.

Theories from Mathematics or Science Education (TMSE)

Slightly less than half of the articles in the review referred to general theories of mathematics research. These general theories are used as background to frame and guide empirical investigations. Generally, they have a descriptive or analytic nature but, often, they also imply general advice for teaching. Cobb and Bauersfeld's (1995) "translation" of interactionism into mathematics education constitutes an example of a theorization belonging to this category.

Theories with a Broader Range on Epistemological Aspects (TEA)

Theories in this category concern learning or cognitive development from a perspective that is not restricted to mathematics or statistics education and that has its origin in another discipline, such as psychology, sociology, or philosophy. Examples are Vygotsky's (1978) *learning theory* and Bourdieu's (1984) understanding of *culture*. Less than a third of the articles referred to TEA.

LOOKING FORWARD ON THEORY DEVELOPMENT IN SER

Our review reveals that SER has begun to mature as a scientific discipline in terms of ToS. Focusing on domain-specific theories about statistics learning is of course important and necessary; it is what gives SER its identity. However, in our review we distinguish five epistemological issues in SER that we consider to be in need of further theoretical elaboration. Next, we report on and discuss these five issues.

The relationship between formal and personal views of statistics

We identify a dilemma in the relationship between formal and personal meanings of statistics. On the one hand, many studies are based on the implicit assumption that teaching should take students' prior experiences and knowledge as a starting point and they attempt to align students' personal conceptions with the target of teaching. However, on the other hand much evidence testifies that students' prior understanding often impacts and stands in conflict with the formal way of understanding key concepts of probability and statistics (Fischbein & Schnarch, 1997; Kahneman, Slovic, & Tversky, 1982). We believe that SER needs to make this dilemma an explicit object of theoretical investigations. There is a need of a theoretical discussion that makes it possible to understand, explain, and predict processes involved in the relationship between personal and formal meanings of statistics. Research needs to answer questions such as: What is the meaning, or possibility, to build on personal ideas when they are at odds with the learning goals? and, How is it possible to align opposing understandings?

Progression in SPcT

In line with the assumption that teaching should build on students' mathematics, SER has come to develop frameworks that describe levels in students' understanding of statistics and probability. This kind of empirical research has emphasized students' personal conceptions of statistical products. Detecting and describing levels of understanding or ability to deal with statistical processes has attracted less attention. More specifically, we identify a lack of theoretical knowledge on how to conceptualize progression, learning trajectories, in frameworks that are used to describe processes involved in statistical investigations such as a) formulating a statistical researchable question, b) modeling, and c) contextual awareness. For instance, say that a class comes up with a number of different formulations of a statistical question to investigate: How can research provide tools for the teacher to conceptualize and distinguish progression of statistical sophistication in the students' questions?

Combinatorial dominance in probability education research

In line with the discussion in Nilsson et al. (2017), we note that a combinatorial perspective dominates research on teaching and learning probability. Understanding *compound random phenomenon* (CRP) is often considered from how students are able to generate complete sets of outcomes, and sample space composition is used as a basis for making probability predictions (Polaki, 2005). Although we endorse that students should understand the role of sample space for making predictions on outcomes of CRP, there might be concepts and principles of relevance for a holistic understanding of CRP that do not get enough attention in such a perspective. For instance, the Product Law of Probability (PLP) does not enter naturally when approaching CPR from a sample space, combinatorial perspective. We are not encouraging research to avoid a combinatorial perspective. What we aim at is to make aware that one perspective on probability modeling dominates SER and that this dominance may limit the theory development on the teaching and learning of CPR.

Theories about technology use

Technology changes the discipline of statistics itself and encourages researchers to rethink learning goals of statistics education (Gould, 2010). However, in our review we did not find clear evidence of that changes in technologies have had impact on rethinking learning goals in the teaching of statistics. We did not find what (diSessa & Cobb, 2004) would describe as frameworks for action or local instruction theories involving or building on new technology. The research motivates and conceptualizes new technology mainly on empirical results, emerging from individual case studies, with limited grounding in theories of learning or teaching. It is hard to note accumulated results or consensuses, except for an overall argumentation of the possibility to provide visualizations, simulations, and different forms of representations by new technology. Except for Watson (2008), the 35 articles reviewed showed rare evidence of theoretical attempts on a more specific level with the intention to provide prescriptive information for supporting statistics learning with technology, such as guiding principles for designing tasks and sequencing tasks in a digital learning environment, or frameworks for explaining and understanding the relationship between digital and analogue learning environments.

The role of context

> When studying reasoning that underpins statistical inference, we have to consider the role of context. (Makar, Bakker, & Ben-Zvi, 2011, p. 155)

We have no reason to question Makar et al.'s (2011) introductory statement. However, based on the review we would like to point out that SER considers context, in relation to "the role of context" (ibid.), from two different epistemological perspectives. On the one hand, context is considered in relation to the *task context*. On the other hand context is considered in relation to norms and affordances of the actual teaching situation, that is, in relation to *the classroom context*. Task context relates to SPcT. It relates to statistical competency/literacy (Gal, 2004) in that it concerns understanding the role of context in statistical reasoning and, particularly, the ability to "shuttling backwards and forwards between thinking in the context sphere and the statistical sphere" (Wild & Pfannkuch, 1999, p. 228). Realizing the difference between training and match in the investigation of the probability of scoring a penalty kick in soccer concerns the ability to understand the role of the task context(s) for the validity of such an investigation. Concerning classroom context, SER highlights issues of teacher knowledge, task-design, and discursive structures of teaching. The teaching perspective on context reflects an increased awareness of the situated and social nature of learning and teaching in mathematics (Lerman, 2006). How teaching is likely to trigger deterministic reasoning rather than probabilistic and data-centered reasoning (Nilsson, Eckert, & Pratt, 2018) can be considered a matter of the role of context from a teaching perspective.

Both perspectives on the role of context are important to take into consideration in SER. However, we argue that, in making the distinction and relation between the two perspectives explicit, research in SER would increase its theoretical focus and enable researchers to build on one another's work even more reliably. An increased theoretical precision will also increase the implementation fidelity of research outcomes into a classroom practice (Lester, 2010).

CONCLUDING REMARKS

In this paper we are looking back and forward on theory use in SER. Based on a literature review presented elsewhere (Nilsson et al., 2017), we described the kinds of theories used in SER as well as to what extent they are used. However, the literature review is a restricted source, given that the number of articles reviewed and the scope of studies were limited. For further discussion of theories in the domain of SER, we hope others feel invited by this modest review to deal with the topic more extensively. For example, a larger set of publications, including theoretical ones, may help to identify further trends. Within mathematics education, theory has been the topic of many publications including books (Sriraman & English, 2010). However, in SER such publications are still practically absent. Promising approaches may be to use one study in statistics education as the source of reflection from different theoretical perspectives (see Bikner-Ahsbahs & Prediger, 2010).

A last point concerns *inferentialism*, which is a theory that is increasingly attracting attention in SER (and other educational disciplines) (Bakker & Hußmann, 2017). Inferentialism is a semantic theory, formulated by the philosopher Brandom (1994), which puts inference at the core of human knowing, and thus fits well with the idea of statistical inference at the heart of statistical knowing.

Bakker and Derry (2011) draw three lessons from inferentialism for statistics education. First, statistical concepts should in their view be primarily understood in inferential terms—that is in their role in reasoning. Trying to move carefully from descriptive to prescriptive ideas, they explicitly take the step from philosophy to education: If, from a philosophical point of view, the inferential role of concepts should be privileged over their representational function, then educators may also need to emphasize the importance of concepts in use. This is the first lesson drawn from inferentialism. The second lesson is that a holistic approach should be prioritized over an atomistic one. Given that concepts only have meaning in relation to other concepts, statistical concepts should be learned in packages—in relation to one another. For example, mean and standard deviation have more meaning in relation to distribution than in isolation. This implies that informal attention to distribution may be needed well before any formal definition can be given. As a third lesson, (Bakker & Derry, 2011) illustrate what privileging an inferentialist approach to teaching statistics may look like being in contrast to a representationalist approach. In this way, they try to link a theoretical background theory on epistemology to didactical ideas about informal inferential reasoning. As their study testified, such theoretical work is far from trivial, but, in our view, necessary.

In our view, inferentialism has the potential to address the previous needs from a fresh perspective. Firstly, it offers a perspicuous view on the *relation between the individual and social* (Schacht & Hußmann, 2015) that underlies the pedagogical challenge formulated in the first need. Schindler and Seidouvy (in press) showed how inferentialism can help to understand how students' informal inferences are socially negotiated in group work, how students' perceived norms influence their informal inferential reasoning (IIR), and what roles statistical concepts play in students' IIR. Secondly, by understanding concepts, categories, and representations in terms of inference and reasoning, the inferentialist language and ways of thinking may well offer the *dynamic and holistic view* that can help to avoid static usage of frameworks with categories or levels (Schindler, Hußmann, Nilsson, & Bakker, 2017). Thirdly, the issue of technology forces scholars to think about the distribution of cognition among humans and machines. Although Brandom's (1994) primary interest is *human* reasoning, the focus on inferences can still offer a fresh perspective on what students need to learn. When using technology, particular inferences are outsourced in computational form to technology but humans still have to decide which technology to use and how to interpret the outcomes. Moreover, doing statistics with technology allows humans to travel fast and far, with the obvious advantage of being able to infer things that are impossible with pen and paper but with the drawback of not necessarily having sight on the route taken (Biehler, 2013).

Finally, we hope that future work in SER will increasingly take into account theories that address epistemological aspects: theories about learning, knowledge, communication, etc. Up to date, only a minority of research articles appears to explicitly address background theories (such as constructivism). However, we found the promising trend that inferentialism, a semantic theory grasping knowing and reasoning both in its individual and social facets, is increasingly used and explicitly discussed and reflected on in SER. We think that such explicit theory use contributes to scientific quality in SER and we hope that future research will continue this path.

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