Tools for Structured Matrix Computations:
Stratifications and Coupled Sylvester Equations

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Abstract
Developing theory, algorithms, and software tools for analyzing matrix pencils whose matrices have various structures are contemporary research problems. Such matrices are often coming from discretizations of systems of differential-algebraic equations. Therefore preserving the structures in the simulations as well as during the analyses of the mathematical models typically means respecting their physical meanings and may be crucial for the applications. This leads to a fast development of structure-preserving methods in numerical linear algebra along with a growing demand for new theories and tools for the analysis of structured matrix pencils, and in particular, an exploration of their behaviour under perturbations. In many cases, the dynamics and characteristics of the underlying physical system are defined by the canonical structure information, i.e. eigenvalues, their multiplicities and Jordan blocks, as well as left and right minimal indices of the associated matrix pencil. Computing canonical structure information is, nevertheless, an ill-posed problem in the sense that small perturbations in the matrices may drastically change the computed information. One approach to investigate such problems is to use the stratification theory for structured matrix pencils. The development of the theory includes constructing stratification (closure hierarchy) graphs of orbits (and bundles) that provide qualitative information for a deeper understanding of how the characteristics of underlying physical systems can change under small perturbations. In turn, for a given system the stratification graphs provide the possibility to identify more degenerate and more generic nearby systems, that may lead to a better system design.

We develop the stratification theory for Fiedler linearizations of general matrix polynomials, skew-symmetric matrix pencils and matrix polynomial linearizations, and system pencils associated with generalized state-space systems. The novel contributions also include theory and software for computing codimensions, various versal deformations, properties of matrix pencils and matrix polynomials, and general solutions of matrix equations. In particular, the need of solving matrix equations motivated the investigation of the existence of a solution, advancing into a general result on consistency of systems of coupled Sylvester-type matrix equations and block-diagonalizations of the associated matrices.

Keywords
Matrix, matrix pencil, matrix polynomial, state-space system, eigenvalue, generalized eigenvalue problem, polynomial eigenvalue problem, perturbation, orbit, bundle, codimension, stratification, matrix equation, Sylvester equation, consistency

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