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# Data generation in statistics – both procedural and conceptual. An inferentialist analysis

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Data generation in statistics education is often conducted by the students themselves; however, the question of what learning opportunities the data generation process offers has only been studied to a small extent. This paper investigates to what extent data generation is an observational and procedural vs. a conceptual activity. We inquire into this question based on an empirical study where eleven year old students measured the jump lengths of paper frogs. Our analysis draws on students' discussions in group work, and it uses inferentialism as a background theory. Our results indicate that students' discussions are conceptual to a certain extent and provide various learning opportunities for the students.

Research in statistics education is organized around three major axes: data generation, data analysis, and statistical inferences (Garfield & Ben-Zvi, 2008). While the importance of data and data generation is widely acknowledged, it has received little attention in research (Garfield & Ben-Zvi, 2004), and there is a tendency in statistics education research to reduce data generation to a means to introducing and investigating other statistical ideas, or to a series of actions and procedures to get the right data (Cobb & McClain, 2004).

In recent decades, these views on data generation began to change. Garfield and Ben-Zvi (2008) argued that how data is obtained is important information about any statistical study. In a similar manner, researchers began to stress the importance of data generation for the conceptual development of statistical ideas (e.g. Cobb & Moore, 1997). Based on empirical findings on how data creation can enhance students' statistical reasoning, Cobb and McClain (2004) recommend that students should produce their own data. Other researchers recommend instead that the data does not necessarily have to be produced by the students themselves as long as it is given authentic context (e.g. Hancock, Kaput & Goldsmith, 1992). These differing views reflect the fact that there is no agreement on the significance of students' involvement in data generation processes for learning about statistical concepts. It is an open question whether

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data generation offers worthwhile learning opportunities for students; whether it is – despite certain opportunities for conceptual development – mainly procedural; and what the conceptual aspects of learning in data generation processes might be.

This paper aims to investigate the data generation process. Based on an empirical study where eleven year old students produced data collaboratively in groups, we investigate the extent to which students' discussions in data generation focus on observation and its procedures or on conceptual aspects of statistics learning. To analyze the conceptual and procedural sides of data generation, we use the semantic theory of inferentialism (Brandom, 1994; 2000). Inferentialism offers us a tool to analyze students' data generation as well as its conceptual and procedural aspects.

## Previous research

Concepts such as data collection, data production, data generation, and data fabrication are used interchangeably in statistics education research, and sometimes the terms are used to describe different processes. For instance, Cobb and McClain (2004) used the term "data generation" to make a clear demarcation from simple data collection. According to these authors, data generation is a process that precedes data collection: "these preceding phases involve clarifying the significance of the phenomenon under investigation, delineating relevant aspects of the phenomenon that should be measured, and considering how they should be measured" (Cobb & McClain, 2004, p. 386).

Reviewing research on the process of data production in teaching/learning situations, we note two major approaches. In the first approach, data is handed to the students via the teacher or textbook, and the students' work with data generation consists of making sense of and contextualizing this data. In most of these studies, the researchers' target is the learning of a specific statistical concept rather than the experience of a statistical investigation (Heaton & Mickelson, 2002; Lehrer & Schauble, 2002). Singer and Willet (1990) argued that teacher-produced data or textbook data should be authentic data and that real data can connect to real context and, as such, enhance interest, relevance, and substantive learning. The underlying conjecture is that authentic contexts will trigger students' engagement, and pave the way for fruitful discussions with data. Cobb and McClain (2004) proposed that when students are handed data, they should be told the story of the data and how the data has been produced. In contrast, Noss, Pozzi and Hoyles (1999) warned that focus on real data can overshadow statistical ideas and give priority to deterministic reasoning. They suggested that, for educational purposes, focus should be put on how concepts in statistics are used in practice. The second approach, student-produced data, has been variously conceptualized in research depending on the researchers'

agenda. In general, the students produce the data either through computer-simulation (Ainley, Pratt & Hansen, 2006; Pratt 1995) or manually in experimentation (Nilsson, 2013). Lehrer and Romberg (1996) take a modeling perspective on data. From their perspective, data construction involves "posing question, collecting response, transforming response into data" (p. 76). The data collected by students may be "treated as objects independent of the existence of that which they represent" (p. 70), and may serve as objects of manipulation for further inferences. Lehrer and Romberg's study indicates that the construction of data enhances students' ability to draw sound inferences beyond the data (Makar & Ben-Zvi, 2011). Nilsson's (2013) study shows that students' involvement in data generation does not automatically improve their prediction ability. In Pratt's (2000) study, the students used a data-simulation (Chance-maker microworld) to produce data.

A trait common to these two approaches is that how data is gathered is often understood basically procedurally (Garfield & Ben-Zvi, 2008). Studies that focus on conceptual aspects and involve student data generation typically discuss how the data generation can be supportive of the learning of statistical concepts; they do not focus on learning opportunities of the data generation process itself. The conceptual learning opportunities in data generation are, therefore, an under-researched topic.

## Theory

In order to understand the conceptual and procedural in data generation practices, we use the semantic theory of inferentialism, as primarily developed by Brandom (1994; 2000). By using concepts from inferentialism, we look in detail at students' practice of measuring and how it relates to the students' social practice in their group work, as well as considering the existing and emerging conceptualization of the situation that comes into play in collaborative work. This is made possible because inferentialism provides detailed accounts of how the social practice of communicating underpins the intellectual practice of being a concept user.

Inferentialism has gained popularity in mathematics and statistics education research in recent years, and it has been used in different ways in order to conceptualize students' understanding and reasoning (Bakker & Derry, 2011; Schindler et al., 2017). A recent special issue on inferentialism in the Mathematics Education Research Journal (Bakker & Hußmann, 2017), including various empirical studies (e.g. Mackrell & Pratt, 2017), gives a glimpse of the potential that inferentialism offers for understanding students' activities in statistics. However, most of the work focuses on students' individual work. Group work has not been analyzed to a great extent using inferentialism (exceptions are Schindler & Joklitschke, 2016; Schindler & Seidouvy, in press).

Inferentialism is a theory that explains how shared conceptual content can come out of interpersonal communication (and asserts that all conceptual content is of this kind); concepts only have meaning in relation to other concepts in inferential networks. In inferentialist theory, knowing a concept is knowing how to use it in communication.

Making explicit how this is possible means explaining how communication can have the necessary interpersonal precision for concepts to develop stable relationships to other concepts. Following the later Wittgenstein (1958), Brandom sees the discursive practice as a language game governed by a set of rules, but looks at this game in a different way by specifying that it is a game of giving and asking for reasons. When using a concept, one commits to its relation to other concepts. Any claim made can both serve as a reason and stand in need of other claims as justification. In this way, interlocutors can give and ask for reasons concerning some content or concept and can develop a shared way of using concepts. Interlocutors can acknowledge another person's claim explicitly, or entitle it by implicitly using it as a premise in further communication.

However, inferentialism is also compatible with the general theorization of communication and all its imprecise messiness, as discussed, for example, by Harvey Sacks (1995). As Münchhausen's trilemma exemplifies, we cannot expect to define anything fully without ending up in infinite regressions, in circular arguments, or relying on unexplained axioms. In this sense, communication would seem to lack the necessary precision to ever allow agreement about any concept's use. Why would interlocutors ever stop asking for further justification? The resolution of this situation comes from acknowledging that language use is not only a logical inferential practice but also a social inferential practice. As social, conscious, and self-conscious beings, humans can decide when it is not appropriate or practical to ask for further reasons or justifications – even in cases where our judgment tells us that it is logically called for. In the game of giving and asking for reasons, in Brandom's formulation, making an assertion means undertaking responsibility to justify the claim if appropriately challenged. If an interlocutor grants another person's claim *authority*, the claim can stand without further challenge. It is in this way that infinite regressions of justification are avoided.

Brandom (1994) discusses three types of authority: person-based authority, content-based authority, and observational authority. Content-based authority is "invoked by justifying the claim through assertion of other sentences from which the claim to be vindicated can appropriately be inferred" (p. 175). For person-based authority, the speaker does not provide content-related reasons herself, but defers to the claim of another person. The speaker reasserts the claim of another person. In this turn, entitlement to the claim is "inherited" from the original speaker who uttered the claim in the first place. The last of Brandom's three authority types, observational authority, is the most interesting to us.

Observational reports are empirical claims. As such, they are a particular type of noninferential reports, which means that they are contentful, but with no explicit inferences to other claims. In contrast to claims that refer a term to a category, such as "five is a number," empirical claims such as "there is a dog outside" require two particular sets of authorities to be in place. First, for anyone to call you a reliable reporter of an observation, they need to hold the belief that you have appropriate conceptual knowledge of the content that you are reporting, for example that you can distinguish dogs from other animals. We will call the authority associated with having conceptual knowledge of the empirical matter to be assessed *empirical content authority*. Second, for anyone to call you a reliable reporter of an observation, they need to hold the belief that you are in the appropriate circumstances to evaluate the empirical matter, for example that you are in fact looking outside. Such appropriate circumstances might involve other matters than physical position and we will here call the authority obtained by being in the right circumstances for a particular observation *circumstantial authority*. Hence, *empirical content authority* and *circumstantial authority* together make up *observational authority*.

From this line of reasoning, we can draw a theoretical conclusion. If the topic is fixed and other interlocutors entitle a person empirical content authority, gaining observational authority will be a matter only of other interlocutors also seeing that the person is in the appropriate circumstance to make the observation. And by extension, if there is a shared understanding that all people in a group have empirical content authority, anyone who gains circumstantial authority will also gain observational authority. Based on the inferentialist account of authority in students' work, we ask the following research questions: 1. What distribution of authority types can we identify in students' work? 2. What, if any, conceptual opportunities to learn can be detected in the students data generation process?

## Method

The data material for the present study consists of video-taped group work sessions with two groups (A and B) of, for each group, four 11 year old students from a grade five class in a Swedish school. According to their teacher, the students were familiar with group work and with sharing the work among themselves, but were not used to conducting experiments themselves. As part of a bigger design research project, the groups were given a task of our design by their teacher. In this task, the students were asked to experiment with paper frog models of three different sizes. By pushing down on the frog with a finger and releasing, the frogs made a small jump. The task was set in a narrative where the students were called in as experts to assist a company in finding "the best-selling frog." This was explained to mean "the frog that jumps the furthest." In

the instructions, the students were asked to collaboratively test and record five jumps for each of the three frogs, and then to come up with a recommendation for the company. The students had measuring tapes and a pre-prepared protocol paper where they could record their measurements.

While the whole lesson was video-taped, here we only deal with the sequences that concern data generation (in the sense of Cobb & McCain, 2004). By using inferentialist theory, we analyze the conceptual and social content that underpins different instances of data generation. In this analysis, inferentialist theory is operationalized as follows. Deciding on which number to write down is for each measurement considered as authorizing an observational report. For each such case, we analyzed the section of the video and corresponding transcripts from the point where the frog was positioned on the starting point to the production of the written note. By analyzing the communicationally relevant acts (speech acts, positions, movements, and gestures), we categorized the claims that were acknowledged in the discussion according to which type of dominant authorization underpinned this acknowledgment, as described in the theory section. Thus, students' interactions leading up to the production of a written number are considered the primary units of analysis.

## Results

The interactions in the two groups followed different patterns. Most of the results that we want to share concern group B, because their work turned out to exemplify particularly interesting phenomena in relation to our research question. We will briefly deal with group A at the end of the results section. Before presenting our general results, we exemplify our analysis by describing group B's collaboration related to some of the jumps.

The first measuring session (jump 1) starts after an initial discussion of the instructions given to the students. The students first discuss the initial positioning of the frog at the starting point, agreeing that the front foot of the frog should be at 0. After the jump, all four students lean forward to look at where the frog landed. One student says, "We measure from here" and points with the pen at a front foot of the frog. Others acknowledge. Several students state "40." Just when one student leans down to write it down, someone says "40 millimeters." This initiates a discussion on whether the unit should indeed be millimeters or centimeters. Here 40 is already acknowledged as a number representing the jump length. Through inferring that 40 mm would mean 4 cm and that the actual jump is rather close to half a meter, they eventually agree that centimeter is the correct unit. In the end, in our theorization, the claim that gets authority is 40 cm. However, leading up to this claim, distinct inferences about the purpose, content, and theory, or the production of the data point, are made salient. Through gestures, actions and speech, an agreement about the starting position of the frog in relation to the tape is made. Similarly, an agreement is reached on

the meaning and choice of the units. Since these discussions are all related to the content of the measurement (a frog’s jump that is to be quantified as a number with a unit by means of a measuring tape), we classify this as content-based authority. The discussion around jump 1 produces a measurement, but also some rules concerning the measurement. Some of them are implicit, that is, not communicated as verbal claims. However, these rules are later acknowledged in action when the students follow them in subsequent jumps.

In the next jump, the students follow the same procedure for setting up the frog, thereby acknowledging the claim about initial positioning. The rules that the students negotiate do not only concern measurement but also other aspects of the data generation. In jump 2, one student, Daniella, claims she is ”a bad jumper” indicating that her frog’s jumps are shorter. This is acknowledged by the others. Later, in jump 6, this becomes important. Now a new frog should be used. Students discuss jump order and then decide that two of the students should do two jumps each, but Daniella only one, because here jumps are shorter. This is followed in the jumps with the last frog (jumps 11–15). We interpret this negotiated turn taking rule as the students’ possible understanding that if one person systematically produces shorter jumps, letting this student do a different number of jumps for different frogs would skew the results.

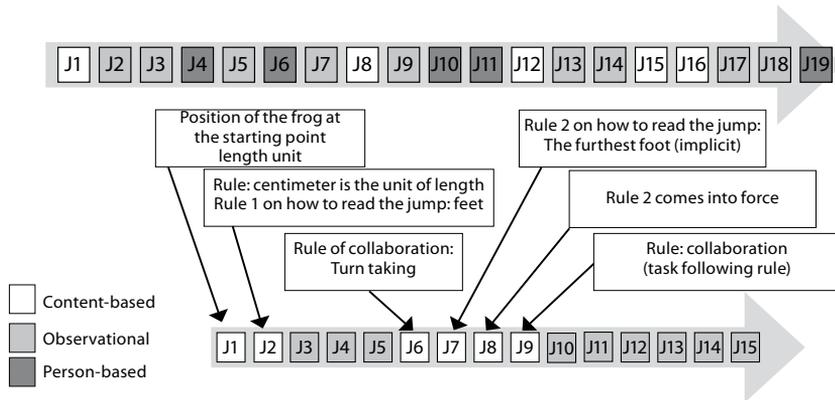


Figure 1. Sequence of authority types for groups A (above) and B

The summary of the analysis of group B is shown in figure 1. In their work, only content-based and observational authority appears. Their work is systematic in the sense that through collaborative work, they make agreements about how to produce the data, in a similar way to that explained for jump 1. Whenever a new jump introduces a new uncertainty of how to measure, a content-based claim that is later acknowledged produces a new rule. As seen, this happens in jumps 1, 2, 6 to 9. Whenever no new uncertainties appear, observational authority takes over. From jump 10 on the students seem to have reached a shared agreement

on how to produce the data. Each jump is dealt with through observational authority. Whichever person is handling the measurement gains observational authority due to circumstantial authority, which is then easy to obtain.

We will now briefly mention also group A. As seen in figure 1, the sequence of authorities plays out very differently here. Group A initially has a rather long discussion about whether millimeters or centimeters are the best units to report. They end up choosing millimeters. This is a discussion concerning the empirical content of the data generation. Group A never agrees on a clear set of rules for the measurement. Lack of such rules means that it is harder to obtain observational authority since there is no shared agreement about what would constitute empirical content authority. And the requirement of measuring millimeters adds to the difficulty of obtaining observational authority. Instead, group A makes choices on what data to report based on content-related claims, which, unlike in group B, do not produce converging agreement about how the measurement should be done. Several of their observational reports are also a product of person-based authority.

## Discussion and conclusion

The purpose of this paper was to investigate to what extent students' discussions in data generation focus on observation and its procedures, or on conceptual aspects of data generation. Based on inferentialist theory, we analyzed students' collaborative work by means of investigating the claims that gained authority in their conversation. Our study revealed that the work of group B provides an example that collaborative work with data production can involve conceptual content (Cobb & McClain, 2004). Defining rules for measuring is in any data generation situation an aspect of giving the numbers context and creating statistical content (Moore, 2006). In group B, instances of content-based authority are followed by observational authority, which is gained by obtaining circumstantial authority. This indicates that the need for rules regarding several aspects of measuring was identified, negotiated, and shared. The fact that rules are both made explicit and integrated as shared assumptions in the subsequent work are an indication of a learning opportunity of this conceptual content.

Group B's collaboration illustrates the possible added value of data generation tasks and activities. Didactically, the task given to the students created a situation that started without clear rules about how the measurements should be done. Such absence of rules seems to be a prerequisite for discussions on the nature of the data generation to take place: If the rules are 100% described, then there is no reason to discuss their conceptual aspects. Yet, group A had few such discussions despite being given the same instructions, and choosing what measurement to write down was often based on person-based authority, frequently reflecting a social negotiation rather than producing and following data generation rules. It is subject to further research to investigate ways of refining task formulations to promote the group B type of experience.

From a theoretical and methodological perspective, viewing our data through the lens of inferentialism enabled us to analyze a negotiation that partly consisted of silent agreements. What enabled us to still draw our conclusions was the role that authority plays in reasoning. With cognitivist theories like constructivism, making the claim we do would mean having to produce ideas about what is in the collaborators minds, because that is by definition where knowledge resides in constructivist theories. Conversation analysis (Sacks, 1995) or other linguistic theories like Toulmin's (1958) model of argumentation could, in similar ways to inferentialism, be used to draw out the type of implicit agreements we deal with here. However, such theories lack the ontological depth of inferentialism where "all questions of ontology are questions on the authority structure in the social practice of claiming" (Wanderer, 2014). In collaborative work the content is, in typical communicational manner, often dealt with implicitly – and not with the explicitness of formal mathematical presentation. We have shown that inferentialism provides a basis for analyzing mathematics conceptualization in such complex communicational settings.

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