REPORT 163

1974

MEASURES OF ATTITUDINAL
POLARIZATION

BO EDVARDSSON

JAN VEGELIUS

DEPARTMENT OF PSYCHOLOGY • UNIVERSITY OF UPPSALA • SWEDEN

MEASURES OF ATTITUDINAL POLARIZATION¹⁾

Bo Edvardsson

Jan Vegelius

University of Uppsala

Abstract

Criteria for measures of attitudinal polarization, i.e., degree of opposition among people on a specific issue, are proposed and some formulas, including the standard deviation, are evaluated in relation to the criteria. The formulas were also tested on empirical data with respect to level, dispersion and agreement of received values. The measures on the whole showed a high degree of agreement. There clearly exist instances where the standard deviation is not an adequate measure of attitudinal polarization. Some guidelines are given for the choice of constant values in some proposed formulas.

Key words: Attitude, measurement, polarization.

1) This research was supported by a grant from the Swedish Council for Social Science Research.

Contents

ABSTRACT	
INTRODUCTION	1
PROPOSED CRITERIA	2
PROPOSED MEASURES AND EVALUATION	5
The standard deviation	6
McCall & Simmons' K measure	6
Francis' G measure	7
Proposals of the present authors	8
Determination of constants	11
EMPIRICAL TEST	13
Method	13
Results	14
DISCUSSION AND CONCLUSIONS	17
REFERENCES	20
TABLES 1 - 6 ,	
FIGURES 1 - 3	
APPENDIX 1:1 - 2	
APPENDIX 2	
APPENDIX 3:1-4 (TABLES A - C)	

Introduction

When studying opinions or attitudes it can be of great interest to have a measure of the degree of attitudinal polarization, i.e., the degree of opposition or conflict among the members of a population. At first thought it may seem that the standard deviation could be used. But it is evident that this measure only handles individual differences with the mean as a reference point, not the degree of opposition among individuals with a neutral standpoint as a reference point. A measure of polarization should in some way make allowance for where the neutral point of a response scale is located.

Some examples illustrating the usefulness of a measure of attitudinal polarization will be given from a specific attitude area. For instance, when investigating attitudes toward environmental problems one can be interested in finding attitude areas where there is a high degree of consensus, e.g., attitudes toward crowding people in large cities, and areas where there is a high degree of dissention, e.g., attitudes toward the use of private cars in cities. One can also be interested in the degree of consensus in different subgroups of a population, e.g., there may be high consensus among environmental activists and low consensus among people in general. A third type of problem would be to describe the development of the attitudes in a given area over time, e.g., the development of attitudes toward economic growth during a ten-year period. Thus a measure of attitudinal polarization can be used to differentiate between attitude areas, population subgroups and time periods with respect to degree of polarization.

McCall & Simmons (1966) proposed a simple measure of attitudinal polarization (commented upon by Francis, 1967 and by Gold, 1967). This measure, however, is not quite satisfactory for certain theoretical reasons. Furthermore, they did not study the degree of agreement between their measure and the conventional standard deviation for empirical data.

The purpose of this report is to propose criteria for evaluation of measures of attitudinal polarization and to use these criteria to evaluate various alternative measures. Furthermore, the measures will be tested on empirical data.

Proposed Criteria

A measure of attitudinal polarization should be able to handle information about how individuals are distributed on a response scale in relation to a neutral point. It will be assumed that the individuals can be classified into an odd number of categories on a bipolar symmetrical response scale, e.g., disagree, undecided, agree. Furthermore, the categories will be assigned integer values from $-k$ to $+k$ under interval scale assumption (cf. Edvardsson & Lindén, 1973). The middle category will be assigned value 0 and will be assumed to represent a neutral attitude.

In order to explicitly specify the task for a measure, one needs a set of criteria, which may then be used for evaluating different proposed measures. Here will be given those criteria which seem to be the most relevant for a measure of attitudinal polarization. Some of the criteria are first stated in a rather vague, general way, after which they are given in a more cogent mathematical way, in which

some of the contents of the general formulations may be lost.

It will be assumed that an index of attitudinal polarization will in some way indicate which side of the response scale (the neutral point excepted) is of lesser weight, i.e., which opinion side is weaker. The side with the lesser weight will be called the "low-weight side."

The following criteria are proposed for an index of attitudinal polarization.

1. Comparability between groups with different numbers of individuals. Mathematical formulation: if the relative distributions of two samples are equal, the index values should be equal.

2. Sensitivity to the position of the response distribution relative to the neutral category, e.g., in Figure 1, distribution b should receive the higher value of polarization. A more restrictive mathematical formulation: if a value changes its sign but not its absolute value and goes from the low-weight side to the high-weight side, the index should decrease.

Insert Figure 1 about here

3. Sensitivity to the dispersion of the response distribution, e.g., in Figure 2, distribution b should receive the higher value of polarization. Mathematical formulation: if a value on any side decreases its absolute value but does not change its sign, the index should not increase. If the low-weight side contains at least one value, the index should

decrease as well.

 Insert Figure 2 about here

4. Sensitivity to the proportion of individuals in the neutral category. Mathematical formulation: if a value is changed from 0 to any other value the index should not decrease. If the low weight side after the change contains at least one value, the index should increase as well.

5. The polarization index should reach maximum value if and only if there is a maximum divergence of attitudes. In order to facilitate comparisons, this maximum value should be equal to a fixed constant. The constant 1 is a convenient choice. A problem arises if the number of individuals in a response distribution is odd. Two formulations of criterion 5 will therefore be given.

5a. The index should be maximal ($=1$) if and only if the two most extreme categories each contain half of the responses. In this case it is impossible to reach the maximum value in a sample with an odd number of individuals.

5b. The index should be maximal ($=1$) if and only if the two most extreme categories contain all of the responses and the difference between the absolute numbers of responses in these categories is at most one. A drawback is that criteria 1 and 5b are not compatible.

6. The index should reach minimum value if and only if the degree of opposition is the smallest possible, i.e., all individuals are of the same opinion. In order to facilitate

comparisons, a fixed minimum value should be used. The value 0 is a suitable choice. Mathematical formulation: the minimum value ($=0$) is taken by the index if and only if no value is negative and/or no value is positive.

7. No response categories should be combined when computing degree of polarization, as this means loss of information, at least when an approximate interval scale is used.

Of the seven criteria mentioned, the mathematical formulation of number 6 is possibly the most disputable. It might be argued that this criterion should be sharpened to require all values in the same category because it can be argued that a distribution within one side of a bipolar response scale can be polarized to a certain small extent.

It is also possible that other criteria can be of importance. One which might seem natural is one that requires comparability between response scales with different numbers of steps with respect to the values of an index of polarization. The authors have preferred not to give such a criterion, as the proportion that chooses the neutral category can theoretically be expected to decrease with an increasing number of response categories. Matell & Jacoby (1972) made an investigation that verified this effect. Even if the number of categories is the same and only the verbal labels differ, the response distribution may be greatly influenced.

Proposed Measures and Evaluation

In this section some proposed measures will be examined in relation to the foregoing criteria.

The standard deviation

The traditional standard deviation satisfies criteria 1, 2, and 7. It also satisfies a stricter version of criterion 6, i.e., it assumes value 0 if and only if all the values are in the same category. It can easily be transformed to satisfy criterion 5a by division by the maximal standard deviation, which can be proved to equal the number of categories on one side of a symmetrical scale (the neutral category excepted). The great deficiency of the standard deviation is that it does not satisfy criterion 4, i.e., when a response is changed from the neutral category to another, the standard deviation may decrease. The reason is that it relates the deviations to the mean, not to the neutral point.

McCall & Simmons' K measure

McCall & Simmons (1966) proposed a measure that is a product of two factors: (a) the intensity factor, defined as the ratio of positives to negatives, with whatever is smaller in the numerator. This factor measures essentially degree of opposition. (b) the extensity factor, defined as the proportion in the sample who have taken a stand on the investigated issue. This factor measures essentially degree of opinion existence.

McCall & Simmons' formula, expressed in absolute frequencies, is

$$K = \frac{n_d}{n_D} \cdot \frac{n_d + n_D}{N} \cdot \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

n_d = the smaller of the frequencies of positives or negatives

n_D = the larger of the frequencies of positives or negatives

N = the total number of individuals in the response distribution

The measure K is not defined in the case when all the individuals are neutral. In that case the intensity factor is of type $\frac{0}{0}$. A natural extra definition is that K is set equal to 0 in this case. This modification is henceforth assumed.

It is now easily realized that the range of variation for K is $0 \leq K \leq 1$, because both factors of the measure obviously have this range of variation.

K satisfies criteria 1, 2, 5a, and 6. Criterion 3 is not applicable, because the measure assumes a trichotomy. Criterion 4 is not satisfied, as may be seen from the following case: the response distribution 1 1 1 (absolute frequencies) gives $K = \frac{2}{3}$, while the more polarized distribution 1 0 2 gives $K = \frac{1}{2}$. Criterion 7 is not satisfied, because the measure presupposes trichotomization of the response distribution into positives, neutrals and negatives. This results in a loss of information if the number of categories is larger than three.

Francis' G measure

Francis (1967) criticized the measure proposed by McCall & Simmons. He instead suggested another measure presupposing items with three categories assigned the values -1, 0 and +1. The formula he proposed is

$$G = \frac{s^2}{(1 - (\bar{x})^2)} \quad \left[2 \right]$$

s^2 = the variance

$\bar{x}$ = the mean

The numerator can be developed.

$$s^2 = \frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2 = \frac{1}{N} \left[\sum_{i=1}^N x_i^2 - N \cdot (\bar{x})^2 \right] = \frac{1}{N} [N_1 + N_{-1} - N \cdot (\bar{x})^2] = [P_1 + P_{-1} - (\bar{x})^2] \quad [3]$$

x_i = observed value for individual number i , i. e. -1, 0 or 1

N = total number of individuals

N_1 = frequency in category with value 1

N_{-1} = frequency in category with value -1

$$P_1 = \frac{N_1}{N}$$

$$P_{-1} = \frac{N_{-1}}{N}$$

$$\text{Thus } G = \frac{P_1 + P_{-1} - (\bar{x})^2}{1 - (\bar{x})^2} \quad [4]$$

This index is not defined if all the values are in category 1 or all are in category -1. It is not possible to give an extra self-evident definition for these cases. For all other cases with no neutrals, the index is 1 ($P_1 + P_{-1} = 1$ in Formula 4). Among the defined cases, the only case when G can be 0 is when all the values are in the neutral category.

From this mathematical reformulation and examination, it follows that G does not satisfy the criteria 2, 5, 6, and 7. G will therefore not be considered further in this report.

Proposals of the present authors

McCall & Simmons' simple formula for trichotomized distributions can be generalized to a formula for any odd number of scale steps with a neutral category in the middle. It is

assumed that the neutral category is assigned the number 0 and that the other categories are assigned positive and negative numbers in a symmetrical manner under interval scale assumptions. The generalization is

$$V_1 = \frac{\min \left(\sum_{i=1}^N |x_i|^C, x_i < 0; \sum_{i=1}^N |x_i|^C, x_i > 0 \right)}{\max \left(\sum_{i=1}^N |x_i|^C, x_i < 0; \sum_{i=1}^N |x_i|^C, x_i > 0 \right)} \cdot \frac{\sum_{i=1}^N |x_i|^D}{N \cdot K^D} \quad [5]$$

If the denominator of the first factor is 0, V_1 is defined = 0.

x_i = observed value for individual i

K = number of categories on each side of the neutral category
(the neutral category excepted)

N = total number of individuals

C, D = positive real constants, the values of which will be discussed later

Using the terminology of McCall & Simmons, this formula, like theirs, consists of an intensity and an extensity factor. Each factor can vary between 0 and 1 and $0 \leq V_1 \leq 1$. In the case when $C = D = 1$, the intensity factor (first factor) can be verbally defined as the absolute amount of deviation from the neutral point on the low-weight side divided by the same amount on the high-weight side. The extensity factor (second factor) can be defined as the received absolute amount of deviation from the neutral point divided by the theoretically maximal amount of deviation, which is the number of individuals multiplied by the number of scale steps on one side of a symmetrical scale.

It is easily verified that McCall & Simmons' formula is a special case of V_1 , namely when only three categories are used ($K = 1$). In that case the values of C and D are irrelevant.

(If one raises the absolute value of plus or minus 1 to any positive value the result will be 1.)

The measure V_1 satisfies the criteria 1, 2, 5a, 6, and 7. Criterion 4 is not satisfied, as may be shown by the example used for the measure K. Criterion 3 is not necessarily satisfied. Another deficiency is that a quotient between the smallest and the largest of two values (cf. the first factor) behaves in an unstable manner, especially when the values are low. Therefore two other measures are proposed which solve this problem.

$$V_2 = \frac{\min \left(\sum_{i=1}^N |x_i|^C, x_i < 0; \sum_{i=1}^N |x_i|^C, x_i > 0 \right)}{\left[\frac{N}{2} \right] \cdot K^C} \cdot \frac{\sum_{i=1}^N |x_i|^D}{N \cdot K^D} \quad [6]$$

$\left[\frac{N}{2} \right] = \text{the integer part of } \frac{N}{2}$

The other symbols are defined as for the measure V_1 .

As can be seen from a comparison with V_1 , only the denominator of the intensity factor has been changed. It has been given the maximal value of the numerator. This definition has also the mathematical advantage that no extra definition is necessary, as the denominator is never 0.

V_2 satisfies the criteria 2, 3, 4, 5b, 6, and 7. Criterion 1 is not satisfied, as will be proved. Consider two samples that have answered a trichotomous item: One sample has the frequency distribution 1 0 2 and the other 2 0 4. The relative distributions are obviously the same. In the first sample V_2 is 1 and in the second $\frac{2}{3}$, from which it follows that criterion 1 is not satisfied. (As mentioned above criteria 1 and 5b can never be simultaneously satisfied for any kind

of measure.) Therefore the third proposed measure is

$$V_3 = \frac{\min \left(\sum_{i=1}^N |x_i|^Q, x_i < 0; \sum_{i=1}^N |x_i|^Q, x_i > 0 \right)}{\frac{N}{2} \cdot K^Q} \cdot \frac{\sum_{i=1}^N |x_i|^D}{N \cdot K^D}, \quad [7]$$

The symbols are defined as for the measure V_1 .

In Appendix 1 it is shown that V_3 satisfies the criteria 1, 2, 3, 4, 5a, 6, and 7. Thus V_3 is the most promising measure in relation to the proposed criteria.

For the special case with trichotomous items, V_2 and V_3 can be reduced to the much simpler

$$V'_2 = \frac{n_d}{\left[\frac{N}{2} \right]} \cdot \frac{n_d + n_D}{N} \quad [8]$$

$$V'_3 = \frac{n_d}{\frac{N}{2}} \cdot \frac{n_d + n_D}{N} \quad [9]$$

The symbols are defined as for the measure K .

Determination of constants

One difficult remaining problem is the determination of the values for the constants C and D . At least four methods are possible: (a) subjective rating of degree of polarization for each distribution within a series of distributions, (b) establishing certain requirements for the parameters of the distributions of index values (or possibly the distributions of the two index factors), (c) calibrating the rate of increase of the index values to the rate of increase of some other common measure, (d) weighting of differences in the scale.

Subjective rating. The distributions in Figure 3 were rated for degree of polarization by the authors. The values of the extreme cases were fixed and reasonable values were given to the other distributions. According to the proof given in Appendix 2 the ratings implied $C + D = 1$.

Insert Figure 3 about here

Distribution requirements. It seems reasonable to require that the median level and the quartile deviation for empirical data not be too low in order to utilize the range of variation of the index. These criteria require a test on empirical data.

Calibration. The rate of increase of the standard deviation and the variance were computed for the distributions in Figure 3. If the index of polarization is to have the same rate of increase as the standard deviation, the constants should be $C + D = 1$. If it is to increase as fast as the variance, the constants should be $C + D = 2$. The proofs are given in Appendix 2.

Scale difference weighting. If changes in all the parts of the scale should be given the same weight, the choice should be $C = D = 1$. For high values of C and D the index is more sensitive to changes in the extreme parts of the scale. For low values of C and D the index is more sensitive to changes in the middle parts of the scale and it can be argued that the concept of polarization means that changes near the neutral point are at least as important as changes in the extreme areas.

Empirical Test

Method

The foregoing measures, except Francis' G, were tested on four samples from empirical investigations.

Sample I. A systematic sample of 229 individuals from the Swedish population answered a questionnaire with 75 attitude statements with a five-step Likert-type response scale (Wholly disagree, Hesitantly disagree, Undecided, Hesitantly agree, Wholly agree). The attitude statements concerned environmental problems (cf. Edvardsson, 1973, statements No. 150 - 176; 201 - 248).

Sample II. A sample of 31 environmental activists answered the same questionnaire as Sample I.

Sample III. A group of students made ratings of the quality of the lectures of a university course on 21 seven-step Osgood-type scales. The number of individuals in the response distributions fluctuated somewhat (80 to 93). The bipolar adjectives were e.g., dull - interesting, effective - ineffective, simple - difficult.

Sample IV. A small sample of 14 to 17 students made ratings of the same kind as Sample III on 22 seven-step Osgood-type scales.

The absolute frequency distributions of the samples I, II and IV are given in Appendix 3 (Tables A - C) and the distributions of Sample III are given in the computation example in Table 1.

In order to study the effect of trichotomization of the response distributions, all distributions were trichotomized into three categories: positive, neutral and negative.

The standard deviation was computed for all the original distributions and the general measures V_1 , V_2 , and V_3 were computed,²⁾ both for the original and the trichotomized distributions. For the original distributions, three different values were used for the constants C and D, namely 2.00, 1.00 and 0.50. Furthermore, the medians and the quartile deviations for all measures were computed for each sample separately (Table 2) and the Spearman rank order correlation (CORALL program; Vegelius, 1974) was computed between all measures in all four samples (Tables 3 - 6).

 Insert Tables 1 - 6 about here

Results

Examples. Table 1 and Tables A - C give examples of computations of V_3 with $C = D = 0.50$. The reader may himself judge whether the index values rank order the distributions in a theoretically meaningful way. It can also be seen from the tables how the two index factors interact.

Some examples of results with respect to item content will be given for Sample I and Sample II. From Table A and B it can be seen that the environmental activists on the whole

2) A FORTRAN computer program named POLARI was written and copies may be obtained from the authors. The program can compute any of the measures V_1 , V_2 or V_3 with any values for the constants C and D. An output example is given in Table 1.

had the lowest degree of polarization. There are, however, a few exceptions, e.g., item 15 ("Industry ignores environmental questions"), where the activists had $V_3 = 0.49$ and the population sample had $V_3 = 0.27$. Another example is item 64 ("Future technology will find substitutes for the natural resources that may possibly be used up"), where the activists had $V_3 = 0.39$ and the population sample had $V_3 = 0.17$. These examples demonstrate the analytical potential of the measure.

Medians. It is obvious that the medians are low for all measures and often very low (Table 2). From the formulas used it can be deduced that for each trichotomized item distribution, V_2 and V_3 will give higher values than V_1 . For all item distributions, it is further clear that the values of V_1 , V_2 , and V_3 will increase when the constants C and D decrease. (V_2 and V_3 will be equal when N is even and very similar when N is odd.) These results will then also be valid for the median of a set of items, a fact that can be seen from Table 2. The table also shows that the trichotomized distributions on the whole give higher median values for V_1 , V_2 , and V_3 than the original distributions.

Dispersions. From Table 1 and Tables A - C it is possible to obtain a rough estimate of the range of variation on empirical data for the measure V_3 with $C = D = 0.50$. The ranges of variation are for Sample I 0.05 - 0.51, for Sample II 0.00 - 0.49, for Sample III 0.04 - 0.31, and for Sample IV 0.00 - 0.22. The measure exceeds 0.50 in only one case.

Table 2 gives data for the dispersion of all the measures in the form of quartile deviations. The values are very low.

In addition, it is obvious that the quartile deviations increase with decreasing constant values.

Agreement between the measures. Tables 3 - 6 give the Spearman rank correlations between the measures. In the correlation matrices, the standard deviation is included as measure No. 1 for purposes of comparison. The agreement between the standard deviation and the other measures is high (about 0.90), but not extremely high, in three samples (I, II, III). In Sample IV the agreement between the standard deviation and the other measures is only moderate (about 0.75). This can be expected from the skew distributions in Sample IV. The first column in the four correlation matrices also shows that the standard deviation correlates better with V_2 and V_3 than with V_1 , both for the trichotomous and for the original distributions.

From the correlation matrices it can also be seen that the correlations between the same measures for trichotomized and original distributions are high and increase when the constants C and D decrease. The mean correlations over the four samples for the lowest constant values are $\bar{r}_{25} = 0.99$, $\bar{r}_{38} = 0.99$, and $\bar{r}_{4,11} = 0.99$, i.e., trichotomization does not matter when $C = D = 0.50$ are used.

Another obvious result is that the correlations between V_2 and V_3 are perfect when the same constant values are used, as may be deduced from the formulas.

The correlations between V_1 on the one hand and V_2/V_3 on the other seem to increase with decreasing constant values

($\bar{r}_{7,10} = 0.92$, $\bar{r}_{69} = 0.95$, and $\bar{r}_{58} = 0.96$ for the correlations between V_1 and V_2).

Finally the correlations within the triads with differences only with respect to constant values (5, 6, 7; 8, 9, 10; 11, 12, 13) are high, but never perfect, in the samples. This constitutes empirical evidence that the rank order of the distributions is changed with respect to degree of polarization, i.e., different constant values imply a somewhat different concept of polarization.

Discussion and Conclusions

The need for a polarization measure conceptually distinct from the standard deviation seems undisputable, even though such a measure necessarily must be correlated with the standard deviation. Criteria for such a measure have been proposed in this report. Of the three proposed general polarization formulas, V_3 best fills the proposed criteria. V_2 and V_3 are mathematically equivalent when the number of individuals is even, and even when the number of individuals is odd, there is a perfect correlation between the measures. The user may choose either of these two formulas, but from a theoretical point of view V_3 is to be preferred.

It should also be pointed out that the two index factors can be used as separate measures or as a two-dimensional index for special analytical purposes.

There are certain problems connected with the use of the polarization measures. The empirical data show that the values and the dispersions of the measures tend to be

numerically low. It may be that the empirical data used are not even crudely representative of attitude research data. From a theoretical point of view it can also be argued that society rests upon consensus between individuals and that the measures should reflect such a consensus by giving low values in general. Issues that are highly controversial will thereby be easily discriminated by their high values. One can for instance also compare this with the common correlation measures, which also in general give low values on empirical data, partly because of the structure of reality. The low values do not, however, constitute a serious problem since they, just as well as high values, can be used to rank order distributions.

As has been shown both theoretically and empirically, there are good reasons for choosing low values for the constants C and D. From the proposed criteria for the choice of constant values, $C = D = 0.50$ or perhaps an even lower value would seem to be suitable. Of course there is nothing in the mathematical structure of the formulas implying equal values for C and D. Some combination of unequal values may be shown to be better in relation to some given theoretical or empirical criteria.

Another problem is the measurement of polarization when there is no neutral category. The proposed measures can also be used in this case, but such a use cannot be recommended. The absence of a neutral category can force a polarization that does not exist.

A further problem is that different distributions are reduced to the same numerical value. This is, however, a problem also connected with e.g., the mean and the standard deviation and with information processing in general. One has to decide whether to accept loss of information in order to abstract more general results from data. The criticism by Gold (1967) of McCall & Simmons' K measure does not therefore seem too serious.

References

- Edvardsson, B. Faktoranalys av attitydpåståenden om miljöproblemen. Uppsala: Uppsala Universitet, Psykologiska Institutionen, 1973. (Work report)
- Edvardsson, B., & Lindén, M. A method for evaluation of metric properties of response scales. Uppsala: University of Uppsala, Department of Psychology, Report 146, 1973.
- Francis, R. G. A response to a proposal for a new measure of attitudinal opposition. Public Opinion Quarterly, 1967, 31, 407 - 413.
- Gold, D. Critical note on a new measure of attitudinal opposition. Public Opinion Quarterly, 1967, 31, 76 - 79.
- Matell, M. S., & Jacoby, J. Is there an optimal number of alternatives for Likert-scale items? Effects of testing time and scale properties. Journal of Applied Psychology, 1972, 56, 506 - 509.
- McCall, G. J., & Simmons, J. L. A new measure of attitudinal opposition. Public Opinion Quarterly, 1966, 30, 271 - 278.
- Vegelius, J. CORALL, a fortran program for correlation measures. Uppsala: University of Uppsala, Department of Psychology, Report 147, 1974.

TABLE 1

Computations of V_3 for the
Distributions of Sample III

CATEGORIES = 7. MEASURE = V_3 . C = 0.50. D = 0.50.

NO	INDEX	I-FACTOR	E-FACTOR	-3	-2	-1	0	1	2	3
1	0.0656	0.0559	0.7543	27	27	15	7	6	1	3
2	0.3130	0.4647	0.6735	7	19	23	6	14	9	0
3	0.2326	0.4034	0.5795	3	12	7	10	26	15	4
4	0.0224	0.0272	0.8234	45	20	13	5	2	0	4
5	0.0422	0.0550	0.7080	26	29	21	4	2	0	0
6	0.2103	0.4329	0.4761	2	10	11	29	15	5	6
7	0.0670	0.0974	0.6974	18	33	19	10	6	1	0
8	0.1112	0.2112	0.5207	12	16	17	25	7	5	1
9	0.3950	0.1155	0.9311	36	35	13	4	2	2	2
10	0.0474	0.0974	0.4264	0	0	7	29	17	10	11
11	0.0925	0.1553	0.5965	1	1	9	16	15	23	11
12	0.0955	0.1427	0.6001	1	4	4	20	19	29	13
13	0.1902	0.3753	0.4301	4	11	18	25	17	8	1
14	0.0552	0.0557	0.8180	0	3	1	3	12	14	31
15	0.1634	0.2149	0.7431	25	29	7	0	8	5	1
16	0.0327	0.0556	0.6046	0	1	3	12	16	28	23
17	0.1332	0.3651	0.5004	4	10	5	26	20	14	3
18	0.1391	0.2643	0.7195	14	31	15	6	5	8	2
19	0.1506	0.2504	0.6012	12	21	17	13	7	0	2
20	0.1272	0.1690	0.7525	26	28	10	7	8	3	0
21	0.0739	0.0974	0.7589	0	2	5	0	13	22	33

TABLE 2

Medians and Quartile Deviations of the Polarization Measures

Measures	Sample I		Sample II		Sample III		Sample IV	
	Md	Qdev	Md	Qdev	Md	Qdev	Md	Qdev
<u>Trichotonized distributions</u>								
2. V_1 with $C = D = \text{arbitrary}$	.19	.14	.07	.08	.16	.08	.00	.04
3. V_2 "	.24	.09	.13	.13	.21	.09	.00	.06
4. V_3 "	.24	.09	.12	.12	.21	.09	.00	.06
<u>Original distributions</u>								
5. V_1 with $C = D = 0.50$	.16	.12	.05	.07	.10	.06	.00	.02
6. V_1 1.00	.14	.10	.05	.06	.06	.05	.00	.02
7. V_1 2.00	.11	.08	.04	.05	.04	.04	.00	.01
8. V_2 0.50	.18	.07	.09	.09	.10	.06	.00	.03
9. V_2 1.00	.15	.06	.07	.08	.06	.03	.00	.02
10. V_2 2.00	.10	.04	.06	.06	.03	.02	.00	.01
11. V_3 0.50	.18	.07	.08	.09	.10	.06	.00	.03
12. V_3 1.00	.15	.06	.07	.07	.06	.03	.00	.02
13. V_3 2.00	.10	.04	.06	.06	.03	.02	.00	.01

TABLE 3

Spearman Rank Correlations between all Measures in Table 2

for Sample I. The Standard Deviation (No. 1) is Added.

	1	2	3	4	5	6	7	8	9	10	11	12
1	0.8644											
2	0.9205	0.9622										
3	0.9205	0.9622	1.0000									
4	0.8852	0.8958	0.9036	0.9636								
5	0.8808	0.9035	0.9424	0.9584	0.9972							
6	0.9025	0.9400	0.9489	0.9680	0.9841	0.9930						
7	0.9423	0.9311	0.9370	0.9370	0.9419	0.9447	0.9500					
8	0.9459	0.9326	0.9457	0.9442	0.9060	0.9164	0.9343	0.9299				
9	0.9078	0.7831	0.9731	0.8731	0.8154	0.8371	0.8740	0.9357	0.9719			
10	0.9423	0.9311	0.9370	0.9370	0.9419	0.9447	0.9500	1.0000	0.9399	0.9357		
11	0.9450	0.9085	0.9593	0.9583	0.9069	0.9164	0.9343	0.9809	1.0000	0.9719	0.9899	
12	0.9078	0.7931	0.8731	0.8731	0.8154	0.8371	0.8740	0.9357	0.9719	1.0000	0.9357	0.9719

TABLE 4

Spearman Rank Correlations between all Measures in Table 2

for Sample II. The Standard Deviation (No. 1) is Added.

	1	2	3	4	5	6	7	8	9	10	11	12
2	0.8931											
3	0.9099	0.9711										
4	0.9105	0.9710	1.0000									
5	0.9093	0.9927	0.9766	0.9767								
6	0.9193	0.9703	0.9805	0.9807	0.9836							
7	0.9044	0.9150	0.9445	0.9451	0.9412	0.9780						
8	0.9261	0.9525	0.9927	0.9929	0.9662	0.9801	0.9562					
9	0.9240	0.8763	0.9458	0.9465	0.9059	0.9451	0.9644	0.9694				
10	0.8847	0.7395	0.9223	0.8334	0.7814	0.8459	0.8908	0.8763	0.9491			
11	0.9261	0.9525	0.9927	0.9929	0.9662	0.9801	0.9562	1.0000	0.9694	0.8763		
12	0.9240	0.8763	0.9458	0.9465	0.9059	0.9451	0.9644	0.9694	1.0000	0.9491	0.9694	
13	0.8847	0.7395	0.8323	0.8334	0.7814	0.8459	0.8908	0.8763	0.9491	1.0000	0.8763	0.9491

TABLE 5

Spearman Rank Correlations between all Measures in Table 2
for Sample III. The Standard Deviation (No. 1) is Added.

1	2	3	4	5	6	7	8	9	10	11	12	
2	0.8325											
3	0.8844	0.9597										
4	0.8909	0.9571	0.9987									
5	0.8532	0.9870	0.9662	0.9688								
6	0.8857	0.9727	0.9792	0.9818	0.9857							
7	0.8458	0.9091	0.9286	0.9364	0.9442	0.9675						
8	0.9130	0.9338	0.9870	0.9909	0.9597	0.9779	0.9506					
9	0.9325	0.8675	0.9481	0.9558	0.9078	0.9429	0.9442	0.9805				
10	0.9052	0.7922	0.8805	0.8948	0.8519	0.8935	0.9299	0.9338	0.9779			
11	0.9130	0.9338	0.9870	0.9909	0.9597	0.9779	0.9506	1.0000	0.9805	0.9336		
12	0.9325	0.8675	0.9481	0.9558	0.9078	0.9429	0.9442	0.9805	1.0000	0.9779	0.9805	
13	0.9052	0.7922	0.8805	0.8948	0.8519	0.8935	0.9299	0.9338	0.9779	1.0000	0.9336	0.9779

TABLE 5

Spearman Rank Correlations between all Measures in Table 2
for Sample IV. The Standard Deviation (No. 1) is Added.

	1	2	3	4	5	6	7	8	9	10	11	12
2	0.6129											
3	0.7449	0.9529										
4	0.7412	0.9515	0.9987									
5	0.6203	0.9973	0.9529	0.9502								
6	0.7128	0.9636	0.9892	0.9879	0.9690							
7	0.7461	0.9152	0.9690	0.9677	0.9219	0.9758						
8	0.7535	0.9542	0.9987	0.9973	0.9556	0.9906	0.9704					
9	0.7770	0.9327	0.9892	0.9865	0.9354	0.9811	0.9892	0.9906				
10	0.7831	0.8882	0.9650	0.9636	0.8936	0.9569	0.9919	0.9663	0.9892			
11	0.7535	0.9542	0.9987	0.9973	0.9556	0.9906	0.9704	1.0000	0.9906	0.9663		
12	0.7782	0.9259	0.9852	0.9838	0.9273	0.9785	0.9919	0.9865	0.9987	0.9919	0.9865	
13	0.7831	0.8882	0.9650	0.9636	0.8936	0.9569	0.9919	0.9663	0.9892	1.0000	0.9663	0.9919

Figure 1. Response distributions with equal dispersions but different positions on a seven-step response scale.

Figure 2. Response distributions with different dispersions on a seven-step response scale.

Figure 3. Ratings of the degree of polarization for a series of distributions on a scale from 0.00 to 1.00. For the sake of simplicity, $N = 2$ individuals can be assumed. (cf. Appendix 2)

Proofs that V_3 fulfils the seven criteria

Criterion 1. Take two samples with the same relative distributions. The number of individuals in the two samples is supposed to be N_1 and N_2 .

$V_3(1)$ = value of V_3 for Sample 1

$V_3(2)$ = value of V_3 for Sample 2

T_1 = numerator of index factor 1

T_2 = numerator of index factor 2

$$V_3(1) = \frac{T_1}{\frac{N_1}{2} \cdot K^C} \cdot \frac{T_2}{N_1 \cdot K^D}$$

$$V_3(2) = \frac{\frac{N_2}{2} \cdot T_1}{N_1 \cdot K^C} \cdot \frac{\frac{N_2}{2} \cdot T_2}{N_2 \cdot K^D} = V_3(1)$$

Criterion 2. The only part that changes is the first numerator, which decreases, causing V_3 to decrease.

Criterion 3. In this case the second numerator will decrease while the first will not increase.

Criterion 4. In the first place it is evident that no numerator can decrease. It is then evident that the second one increases and, if the first one is not zero, the whole index will increase.

Criterion 5a. The index is 1 if and only if both factors are 1. The first factor is 1 if and only if the minimum value is equal to $\frac{N}{2} \cdot K^C$ which happens if and only if each of the two most extreme categories each contain half of the values. The second factor is 1 if and only if the two most extreme categories contain all the values.

Criterion 6. V_3 is 0 if and only if at least one of the factors is 0. When the second factor is 0 the first factor will also be 0. Therefore it is sufficient to consider the first factor, which is 0 if and only if there is one side with no values at all.

Criterion 7. Self-evident.

Synchronization of the rate of increase for V_3 with ratings, standard deviation and variance

Consider the five distributions (a) - (e) in Figure 3, The means for all distributions are 0 and $N = 2$. The standard deviations and variance of the distributions are then easily computed by their conventional formulas. The standard deviations are 0, 1, 2, 3, 4. The variances are 0, 1, 4, 9, 16.

The standard deviation has the same rate of increase as the ratings in Figure 3. There is of course no logical reason for this equality. If V_3 is to have the same rate of increase as the ratings and the standard deviation, the following equations will be true.

$$V_3(a) = \frac{0^C}{1 \cdot 4^C} \cdot \frac{2 \cdot 0^D}{2 \cdot 4^D} = \left(\frac{0}{4}\right)^{C+D} = 0$$

$$V_3(b) = \frac{1^C}{1 \cdot 4^C} \cdot \frac{2 \cdot 1^D}{2 \cdot 4^D} = \left(\frac{1}{4}\right)^{C+D} = \frac{1}{4} \iff C + D = 1$$

$$V_3(c) = \frac{2^C}{1 \cdot 4^C} \cdot \frac{2 \cdot 2^D}{2 \cdot 4^D} = \left(\frac{1}{2}\right)^{C+D} = \frac{1}{2} \iff C + D = 1$$

$$V_3(d) = \frac{3^C}{1 \cdot 4^C} \cdot \frac{2 \cdot 3^D}{2 \cdot 4^D} = \left(\frac{3}{4}\right)^{C+D} = \frac{3}{4} \iff C + D = 1$$

$$V_3(e) = \frac{4^C}{1 \cdot 4^C} \cdot \frac{2 \cdot 4^D}{2 \cdot 4^D} = 1^{C+D} = 1$$

Results. $C + D = 1$ gives the same rate of increase as the ratings and the standard deviation. If the values corresponding to the rate of increase of the variance (i.e., $0, \frac{1}{16}, \frac{4}{16}, \frac{9}{16}, 1$) are put into the equations in the same way, the equations are true for $C + D = 2$.

TABLE A

Computations of V_3 for the Distributions of Sample ICATEGORIES = 5. MEASURE = V_3 . C = 0.50. D = 0.50.

NO	INDEX	I-FACTOR	E-FACTOR	2	1	0	1	2
1	0.3004	0.3828	0.7845	114	30	35	19	31
2	0.1768	0.1914	0.9236	17	7	9	22	174
3	0.0882	0.1173	0.7513	140	26	47	9	8
4	0.1178	0.1297	0.9085	12	5	11	29	172
5	0.2509	0.3397	0.7388	27	17	44	37	104
6	0.4095	0.5052	0.6766	62	32	48	57	29
7	0.1396	0.1544	0.9041	9	13	12	21	174
8	0.2021	0.3767	0.5264	27	23	28	39	52
9	0.3036	0.5090	0.6571	41	40	51	54	43
10	0.1446	0.2347	0.6163	87	38	70	23	11
11	0.2040	0.2594	0.7866	118	44	31	17	18
12	0.1814	0.2285	0.7641	14	18	29	44	124
13	0.1522	0.1376	0.7702	12	16	23	51	117
14	0.2025	0.3520	0.5753	61	43	75	33	17
15	0.2773	0.4632	0.5988	44	56	62	46	21
16	0.2720	0.4323	0.6292	25	35	65	33	71
17	0.1736	0.3458	0.5020	24	23	95	42	45
18	0.1087	0.2285	0.4758	53	42	101	23	10
19	0.2232	0.4014	0.5561	49	45	75	46	14
20	0.1147	0.1914	0.5991	11	16	71	55	76
21	0.1089	0.1667	0.6530	97	47	61	16	8
22	0.2563	0.4261	0.6015	26	33	69	43	58
23	0.2058	0.3088	0.6665	23	18	50	72	66
24	0.1051	0.1297	0.8103	6	13	25	50	135
25	0.0895	0.1050	0.8527	174	13	27	10	5
26	0.4588	0.6299	0.7283	77	24	42	45	41
27	0.1074	0.1235	0.8697	9	9	19	29	164
28	0.3194	0.4693	0.6806	35	27	50	52	65
29	0.5093	0.6917	0.7363	69	29	39	45	48
30	0.1229	0.1482	0.8291	9	12	28	26	154
31	0.1413	0.2594	0.5440	64	43	84	26	12
32	0.0770	0.0865	0.8908	5	7	13	34	170
33	0.1269	0.1544	0.8218	9	13	25	41	141
34	0.0915	0.1297	0.7057	7	12	51	44	115
35	0.1745	0.2408	0.7245	13	21	42	51	102
36	0.1911	0.2779	0.6975	21	16	49	61	82
37	0.0466	0.0494	0.9428	3	4	9	10	203
38	0.0953	0.1112	0.8617	8	7	22	26	166
39	0.1175	0.1606	0.7318	128	29	50	10	12
40	0.1213	0.1667	0.7277	124	33	48	16	8
41	0.0590	0.0679	0.8689	178	18	23	6	4
42	0.2201	0.2964	0.7730	120	32	35	26	16
43	0.1491	0.1544	0.5659	17	1	4	12	195
44	0.1528	0.1606	0.5518	17	2	4	22	184
45	0.0946	0.1173	0.8062	154	24	35	8	8
46	0.3414	0.4261	0.8012	35	20	30	33	111
47	0.0843	0.1050	0.8029	151	29	34	9	6
48	0.2342	0.3211	0.7292	28	13	50	28	110
49	0.1757	0.2532	0.6938	22	10	59	28	110
50	0.2050	0.2903	0.7062	24	14	50	45	96
51	0.0609	0.0679	0.8961	7	2	15	28	177
52	0.2614	0.3705	0.7054	99	29	54	18	30
53	0.3109	0.5249	0.5924	57	26	79	23	44
54	0.4671	0.5793	0.6877	60	27	56	26	60
55	0.3827	0.6052	0.6324	58	24	71	21	55
56	0.3188	0.4879	0.6535	74	27	65	22	41
57	0.1133	0.1606	0.7056	13	8	56	31	121

TABLE A (continued)

58	0.4161	0.6176	0.6738	52	27	58	30	62
59	0.2822	0.4199	0.6720	40	12	59	43	75
60	0.2178	0.4446	0.4899	40	30	101	24	34
61	0.3089	0.5373	0.5749	57	18	83	31	40
62	0.2078	0.3644	0.5703	31	16	82	40	60
63	0.2012	0.2841	0.7082	25	11	51	43	99
64	0.1694	0.2903	0.5837	22	16	76	50	65
65	0.1240	0.1420	0.8730	11	2	20	23	167
66	0.0684	0.0893	0.8517	9	1	24	33	162
67	0.1078	0.2161	0.4987	15	14	94	57	49
68	0.1352	0.2478	0.5612	14	20	75	67	53
69	0.0789	0.0926	0.9517	8	4	24	30	163
70	0.1412	0.2161	0.6534	18	10	58	63	80
71	0.0906	0.1914	0.4731	9	19	94	72	35
72	0.2520	0.4446	0.5668	49	42	79	27	32
73	0.2157	0.4199	0.5136	36	18	90	55	30
74	0.1776	0.3211	0.5529	18	27	81	46	57
75	0.2991	0.5183	0.5766	32	39	75	36	47

TABLE B

Computations of V_3 for the Distributions of Sample IICATEGORIES = 5. MEASURE = V_3 . C = 0.50. D = 0.50.

NO	INDEX	I-FACTOR	E-FACTOR	-2	-1	0	1	2
1	0.1773	0.1825	0.9717	25	3	0	0	3
2	0.0912	0.0912	1.0000	2	0	0	0	29
3	0.0854	0.0912	0.9355	27	0	2	0	2
4	0.0448	0.0456	0.9811	1	0	0	2	28
5	0.0566	0.0912	0.6207	1	1	10	5	14
6	0.2402	0.3193	0.7521	11	10	3	6	1
7	0.0420	0.0456	0.9205	0	1	1	4	25
8	0.1234	0.3193	0.6057	3	3	9	8	8
9	0.2355	0.4106	0.5735	8	4	10	7	2
10	0.0840	0.0912	0.9205	25	3	1	2	0
11	0.0	0.0	0.9717	28	3	0	0	0
12	0.1199	0.1369	0.8693	2	1	2	6	20
13	0.1146	0.1369	0.8371	2	1	3	6	19
14	0.0662	0.0912	0.7253	14	10	5	2	0
15	0.4853	0.6387	0.7599	3	10	1	12	5
16	0.2914	0.3650	0.7710	2	6	3	8	12
17	0.1961	0.2281	0.8599	4	0	2	8	17
18	0.0388	0.0456	0.8504	19	9	2	0	1
19	0.0587	0.0912	0.6425	15	5	9	2	0
20	0.0401	0.0456	0.8789	1	0	2	6	22
21	0.0895	0.0912	0.9811	27	2	0	0	2
22	0.1213	0.1825	0.6647	2	2	6	13	8
23	0.1177	0.1369	0.8599	2	1	2	7	19
24	0.0435	0.0456	0.9528	1	0	0	5	25
25	0.0452	0.0456	0.9906	29	1	0	0	1
26	0.0726	0.0912	0.7954	19	6	4	2	0
27	0.0	0.0	0.9583	0	0	1	1	29
28	0.2862	0.4106	0.6970	4	4	5	11	7
29	0.1897	0.2281	0.8316	18	6	2	5	0
30	0.0805	0.0912	0.8827	1	1	1	8	20

TABLE B (continued)

31	0.0855	0.1369	0.6246	13	6	9	3	0
32	0.0895	0.0912	0.9811	1	1	0	1	28
33	0.0	0.0	0.9811	0	0	0	2	29
34	0.0	0.0	0.8922	0	0	1	8	22
35	0.0691	0.0912	0.7576	1	1	4	11	14
36	0.0371	0.0456	0.8127	0	1	2	12	16
37	0.0	0.0	1.0000	0	0	0	0	31
38	0.0420	0.0456	0.9394	1	0	1	3	26
39	0.0	0.0	0.9488	29	2	1	0	0
40	0.1247	0.1369	0.9111	22	5	1	1	2
41	0.0	0.0	0.8198	24	2	5	0	0
42	0.2505	0.4562	0.5491	4	5	9	12	1
43	0.0	0.0	0.9717	0	0	0	3	28
44	0.0	0.0	0.9906	0	0	0	1	30
45	0.0	0.0	0.6719	18	4	9	0	0
46	0.0674	0.0912	0.7387	0	2	4	12	13
47	0.0	0.0	0.9150	22	0	0	0	0
48	0.1229	0.1369	0.8977	23	3	2	1	2
49	0.0	0.0	0.9906	0	0	0	1	30
50	0.0448	0.0456	0.9811	1	0	0	2	28
51	0.0	0.0	1.0000	0	0	0	0	31
52	0.1655	0.1825	0.9071	23	3	2	0	3
53	0.2545	0.2737	0.9299	3	2	1	2	23
54	0.0	0.0	0.9811	29	2	0	0	0
55	0.2520	0.2737	0.9205	3	2	1	3	22
56	0.1273	0.1369	0.9299	2	1	1	3	24
57	0.0	0.0	0.9906	0	0	0	1	30
58	0.0857	0.0912	0.9394	25	3	1	0	2
59	0.1427	0.1825	0.7820	3	0	5	6	17
60	0.0911	0.1369	0.5924	12	6	10	3	0
61	0.0824	0.1369	0.6018	13	5	10	3	0
62	0.0828	0.0912	0.9071	2	0	2	3	24
63	0.0	0.0	0.9394	0	0	1	3	27
64	0.3867	0.5474	0.7064	9	6	5	8	3
65	0.0	0.0	0.9717	0	0	0	3	28
66	0.0	0.0	0.9111	0	0	1	6	24
67	0.1553	0.3650	0.4256	2	7	14	6	2
68	0.2566	0.4562	0.5624	2	8	8	11	2
69	0.0	0.0	0.9583	0	0	1	1	29
70	0.1737	0.2281	0.7615	1	4	3	11	12
71	0.2505	0.4562	0.5491	4	8	9	9	1
72	0.3669	0.5474	0.6703	6	4	7	7	7
73	0.2965	0.5018	0.5908	5	7	8	9	2
74	0.0961	0.1369	0.7025	2	1	6	10	12
75	0.2464	0.4106	0.6002	3	5	8	10	5

TABLE C

Computations of V_3 for the
Distributions of Sample IV

CATEGORIES = 7. MEASURE = V_3 . C = 0.50. D = 0.50.

NO	INDEX	I-FACTOR	E-FACTOR	-3	-2	-1	0	1	2	3
1	0.0	0.0	0.8499	7	7	3	0	0	0	3
2	0.2194	0.4124	0.5320	0	2	3	4	0	5	0
3	0.0877	0.1540	0.5698	0	0	2	3	4	5	0
4	0.0	0.0	0.8273	8	5	2	1	0	0	1
5	0.0	0.0	0.7050	5	4	4	2	0	0	0
6	0.0136	0.0825	0.1650	0	0	3	10	1	0	0
7	0.1243	0.1650	0.7533	3	5	4	0	1	0	0
8	0.0437	0.0825	0.5300	3	4	1	5	2	0	0
9	0.0	0.0	0.8197	6	6	1	1	1	0	0
10	0.0	0.0	0.4455	0	0	1	6	2	5	1
11	0.0	0.0	0.3399	0	0	0	7	4	3	0
12	0.0	0.0	0.7502	0	0	0	1	2	0	1
13	0.1204	0.2474	0.4868	1	3	2	15	2	2	2
14	0.0	0.0	0.8335	0	0	0	0	1	2	0
15	0.0619	0.0722	0.8572	7	7	0	0	0	9	4
16	0.0	0.0	0.8054	0	0	0	1	3	0	1
17	0.0	0.0	0.3624	0	0	0	6	8	9	3
18	0.0454	0.0722	0.6293	3	2	8	2	0	1	0
19	0.0400	0.0770	0.5190	0	6	4	4	1	1	0
20	0.0	0.0	0.8421	6	6	3	0	0	0	0
21	0.0	0.0	0.4984	0	0	0	0	1	0	2
22	0.0507	0.0770	0.6580	0	1	0	3	2	6	3