

Navigation of mobile robots by potential field methods and market-based optimization

Rainer Palm* Abdelbaki Bouguerra†

*AASS, Dept. of Technology, Örebro University, Sweden

†AASS, Dept. of Technology, Örebro University, Sweden

Abstract—Mobile robots play an increasing role in everyday life, be it for industrial purposes, military missions, or for health care and for the support of handicapped people. A prominent aspect is the multi-robot planning, and autonomous navigation of a team of mobile robots, especially the avoidance of static and dynamic obstacles. The present paper deals with obstacle avoidance using artificial potential fields and selected traffic rules. As a novelty, the potential field method is enhanced by a decentralized market-based optimization (MBO) between competing potential fields of mobile robots. Some potential fields are strengthened and others are weakened depending on the local situation. In addition to that, circular potential fields are 'deformed' by using fuzzy rules to avoid an undesired behavior of a robot in the vicinity of obstacles.

I. INTRODUCTION

A group of autonomously acting mobile robots (mobile platforms) is a system of subsystems with different goals and velocities, competing navigation commands and obstacles to be avoided. In the last two decades several methods addressing these issues have been reported. One of the most popular methods for obstacle avoidance is the artificial potential field method [1]. Borenstein and Koren gave a critical view on this method addressing its advantages and drawbacks regarding stability and deadlocks [2]. Aircardi and Baglietto addressed team building among mobile robots sharing the same task and the appropriate decentralized control [3]. In approaching situations robots act as moving obstacles where coordination is done by online local path planning using the so-called *via points*. Further research results regarding navigation of non-holonomic mobile robots can be found in [4] and [5]. The execution of robot tasks based on semantic domain-knowledge is reported in detail in [6].

Trying to achieve different tasks at the same time makes a decentralized optimization necessary. Decentralized methods like multi-agent control are expected to handle optimization tasks for systems being composed of a large number of complex local systems more efficiently than centralized approaches. One example is the flow control of mobile platforms in a manufacturing plant using intelligent agents [7]. Other decentral control strategies are the so-called utility approach [8] and the behavioral approach [9] used for mobile robot navigation. The most difficult task to optimize a decentralized system consisting of many local systems leads us to game theoretic and related methods. One of the most interesting and promising approaches to large decentralized systems is the market-based (MB) optimization. MB algorithms imitate

the behavior of economic systems in which producer and consumer agents both compete and cooperate on a market of commodities. This simultaneous cooperation and competition of agents is also called cooptation [10]. In [11] an overview on MB multi-robot coordination is presented, which is based on bidding processes. The method deals with motion planning, task allocation and team cooperation, whereas obstacles are not considered. General ideas and some results of MB control strategies are presented in [12] and [13]. In [14], a more detailed description of the optimization algorithm is presented. The authors show how to optimize distributed systems by so-called producer and consumer agents using local cost functions. Given desired setpoints and, with this, a cost function for each local system a set of controls has to be found that leads the whole set of local systems to a so-called Pareto-optimum.

The present paper adopts many ideas from [13], [14] and [15] to improve the performance of safe navigation of multiple robots using potential fields. In the context of MB navigation, combinations of competing tasks that should be optimized can be manifold, for example the presence of a traffic rule and the necessity for avoiding an obstacle at the same time. Another case is the accidental meeting of more than two robots within a small area. This requires a certain minimum distance between the robots and appropriate (smooth) manoeuvres to keep stability of trajectories to be tracked. The present paper addresses exactly this point where - as a novelty - optimization takes place between "competing" potential fields of mobile robots, whereas some potential fields are strengthened and some are weakened depending on the local situation. Repulsive forces both between robots and between robots and obstacles are computed under the assumption of circular force fields meaning that forces are computed between the centers of mass of the objects considered. It should be mentioned that each mobile robot uses only local data available through its own sensors in order to compute its local actions. Section II shows the navigation principles applied to the robot task. In Section III the general task of obstacle avoidance for a multi-robot system using artificial potential fields is outlined. Section IV gives an introduction to a special MB approach used in this paper. The connection between the MB approach and the system to be controlled is outlined in Section V. Section VI shows simulation experiments and Section VII draws conclusions and highlights future work.

II. NAVIGATION PRINCIPLES

To illustrate the navigation problems, let, in a working area, n mobile platforms (autonomous mobile robots) perform special tasks like loading materials from a starting station, bringing them to a target station and unloading the materials there. The task of the platforms is to reach their targets while avoiding other platforms according to specific rules. The problem is to find an appropriate navigation strategy.

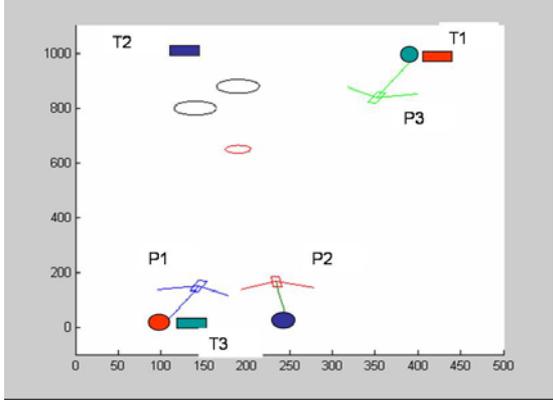


Fig. 1. Platform area

Let, as an example, mobile robots (platforms) P_1 , P_2 , and P_3 be supposed to move to targets T_1 , T_2 , and T_3 , respectively, whereas collisions should be avoided.

Each platform has some estimation about its own position and orientation and also the position of its own target. The position of another platform P_j relative to P_i can be measured if it lies within the sensor cone of P_i .

Navigation principles for a mobile robot (platform) P_i are meant to be *heuristic rules* to perform a specific task under certain restrictions originating from the environment, obstacles O_j , and other robots P_j . As already pointed out, each platform P_i is supposed to have an estimation about position/orientation of itself and the target T_i . This information can be either given in the base frame (world coordinate system) or in the local frame of platform P_i . In our case these information is given in world coordinates. Apart from the *heading to target movement*, all other navigation actions take place in the local coordinate system of platform P_i . The positions of obstacles (static or dynamic) O_j or other platforms P_j are formulated in the local frame of platform P_i .

In the following, 4 navigation principles are formulated that have been used in our work:

1. Move in direction of target T_i
2. Avoid an obstacle O_j (static or dynamic) if it appears in the sensor cone at a certain distance. Always orient platform in direction of motion
3. Decrease speed if dynamic (moving) obstacle O_j comes from the right
4. Move to the right if the obstacle angles β [16] of two approaching platforms are small (e.g. $\beta < 10$) (see Fig. 2)

III. NAVIGATION AND OBSTACLE AVOIDANCE USING POTENTIAL FIELDS

This section gives a more detailed view on the potential field method for obstacle avoidance. Let us start with a simple first order dynamics for each platform P_i which automatically avoids abrupt changes in the orientation

$$\dot{v}_i = k_{di}(v_i - v_{di}) \quad (1)$$

$v_i \in \mathbb{R}^2$ - actual velocity vector of platform P_i ,

$v_{di} \in \mathbb{R}^2$ - vector of desired velocity of platform P_i ,

$k_{di} \in \mathbb{R}^{2 \times 2}$ - damping matrix (diagonal)

The tracking velocity is designed as a control term

$$v_{ti} = k_{ti}(x_i - x_{ti}) \quad (2)$$

$x_i \in \mathbb{R}^2$ - position of platform P_i ,

$x_{ti} \in \mathbb{R}^2$ - position of target T_i ,

$k_{ti} \in \mathbb{R}^{2 \times 2}$ - gain matrix (diagonal)

Virtual repulsive forces appear between platform P_i and obstacle O_j from which 'repulsive velocities' are derived

$$v_{ij_o} = -c_{ij_o}(x_i - x_{j_o})d_{ij_o}^{-2} \quad (3)$$

$v_{ij_o} \in \mathbb{R}^2$ - repulsive velocity between platform P_i and obstacle O_j ,

$x_{j_o} \in \mathbb{R}^2$ - position of obstacle O_j ,

$d_{ij_o} \in \mathbb{R}$ - Euclidian distance between platform P_i and obstacle O_j ,

$c_{ij_o} \in \mathbb{R}^{2 \times 2}$ - gain matrix (diagonal)

Virtual repulsive forces also appear between platforms P_i and P_j from which we get the repulsive velocities

$$v_{ij_p} = -c_{ij_p}(x_i - x_j)d_{ij_p}^{-2} \quad (4)$$

$v_{ij_p} \in \mathbb{R}^2$ - repulsive velocity between platforms P_i and P_j ,

$x_i \in \mathbb{R}^2$ - position of platform P_i ,

$x_j \in \mathbb{R}^2$ - position of platform P_j ,

$d_{ij_p} \in \mathbb{R}$ - Euclidian distance between platforms P_i and P_j ,

$c_{ij_p} \in \mathbb{R}^{2 \times 2}$ - gain matrix (diagonal)

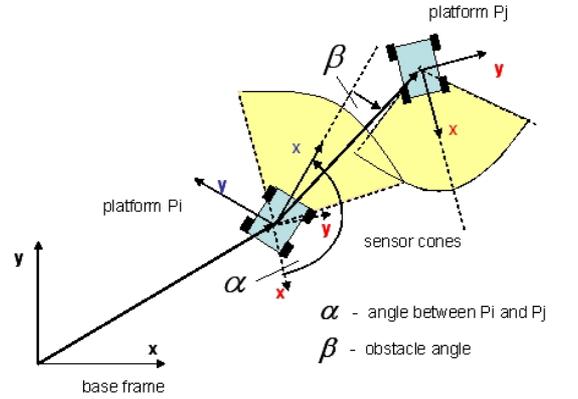


Fig. 2. Geometrical relationship between platforms

The resulting velocity v_{di} is the sum

$$v_{di} = v_{ti} + \sum_{j=1}^{m_o} v_{ij_o} + \sum_{j=1}^{m_p} v_{ij_p} \quad (5)$$

where m_o and m_p are the numbers of contributing obstacles and platforms, respectively. It has to be stressed that force fields are switched on/off according to the actual scenario: distance between interacting systems, state of activation according to the sensor cones of the platforms, positions and velocities of platforms w.r.t. to targets, obstacles and other platforms. All calculations of the velocity components (1)-(5), angles and sensor cones are formulated in the local coordinate systems of the platforms (see Fig. 2)

A. "Deformation" of potential fields using fuzzy rules

Potential fields of obstacles (static and dynamic) act normally independently of the attractive force of the target. This may cause unnecessary repulsive forces especially in the case when the platform can "see" the target.

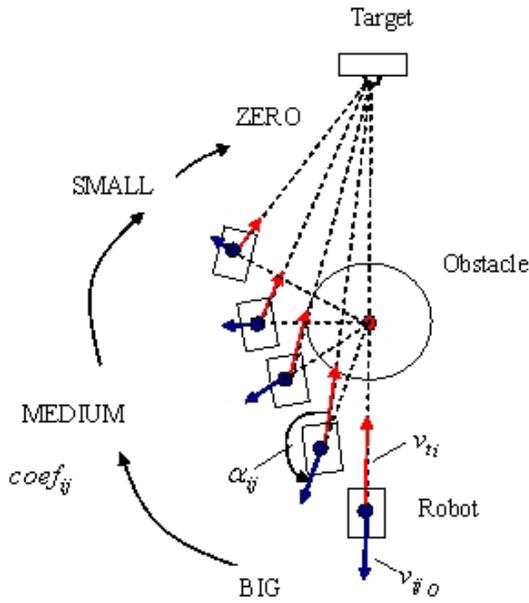


Fig. 3. Deformation of potential field

α_{ij}	Z	S	M	B
B	Z	S	M	B
M	Z	S	M	M
S	Z	M	M	S
Z	Z	Z	Z	Z

Fig. 4. Fuzzy table for potential field

The goal is therefore to "deform" the repulsive potential field in such a way that it is strong if the obstacle hides the target and weak if the target "can be seen" from the platform. In addition, the potential field should also be strong for a high tracking velocity and weak for a small one (see Fig. 3). This is done by a coefficient $coef_{ij} \in [0, 1]$ that is multiplied by v_{ij_o} to obtain a new v_{ij_o} as follows

$$v_{ij_o} = -coef_{ij}c_{ij_o}(x_i - x_{j_o})d_{ij_o}^{-2} \quad (6)$$

The coefficients $coef_{ij}$ can be calculated by a set of 16 fuzzy rules like

$$IF \ v_{ti} = B \ AND \ \alpha_{ij} = M \ THEN \ coef_{ij} = M \quad (7)$$

where α_{ij} is the angle between v_{ij_o} and v_{ti} . The whole set of 16 rules can be summarized in a table shown in Fig. 4. Z - ZERO, S - SMALL, M - MEDIUM, B - BIG are fuzzy sets [17].

IV. MB APPROACH

Imitation of economical market mechanisms and the application to a multi robot system requires the modeling of both the system to be optimized (see Sect. III) and the optimization strategy itself. The system and optimization strategy are presented as continuous models where the computational realization is usually discrete. The desired motion of platform P_i is described by

$$v_{d_i} = v_{o_i} + \sum_{j=1, i \neq j}^m w_{ij}v_{ij_p} \quad (8)$$

where v_{o_i} is a combination of

- tracking velocity depending on distance between platform i and goal i
- repulsive/control terms between platform i and obstacles
- Traffic rules

m - number of platforms

v_{ij_p} - repulsive velocity between platforms i and j

w_{ij} - weighting factors for repulsive velocities where $\sum_{j=1, i \neq j}^m w_{ij} = 1$

The objective is to change the weights w_{ij} so that all contributing platforms show a smooth dynamical behavior in the process of avoiding each other. One possible option for tuning the weights w_{ij} is to find a global optimum over all contributing platforms. This, however, is rather difficult especially in the case of many interacting platforms. Therefore a multi-agent approach has been preferred. The determination of the weights is done by producer-consumer agent pairs in a MB scenario that is presented in the following. Assume that to every local system S_i (platform) belongs a set of m producer agents $Pa_{g_{ij}}$ and m consumer agents $Ca_{g_{ij}}$. Producer and consumer agents sell and buy, respectively, the weights w_{ij} on the basis of a common price p_i . Producer agents $Pa_{g_{ij}}$ supply weights w_{ij_p} and try to maximize specific local profit functions ρ_{ij} where "local" means "belonging to system S_i ". On the other hand, consumer agents $Ca_{g_{ij}}$ demand for weights w_{ij_c} from the producer agents and try to maximize specific local utility functions U_{ij} . The whole "economy" is in equilibrium as the sum over all supplied weights w_{ij_p} is equal to the sum over all utilized weights w_{ij_c} .

$$\sum_{j=1}^m w_{ij_p}(p_i) = \sum_{j=1}^m w_{ij_c}(p_i) \quad (9)$$

The trade between the producer and consumer agents is based on the definition of cost functions for each type of agent. We define a local utility function for the consumer agent Ca_{ij}

$$\begin{aligned} \text{Utility} &= \text{benefit} - \text{expenditure} \\ U_{ij} &= \tilde{b}_{ij}w_{ij_c} - \tilde{c}_{ij}p_i(w_{ij_c})^2 \end{aligned} \quad (10)$$

where $\tilde{b}_{ij}, \tilde{c}_{ij} \geq 0$, $p_i \geq 0$. Furthermore a local profit function is defined for the producer agent Pa_{ij}

$$\begin{aligned} \text{profit} &= \text{income} - \text{costs} \\ \rho_{ij} &= g_{ij}p_i(w_{ij_p}) - e_{ij}(w_{ij_p})^2 \end{aligned} \quad (11)$$

where $g_{ij}, e_{ij} \geq 0$ are free parameters which determine the average price level. It has to be stressed that both cost functions (10) and (11) use the same price p_i on the basis of which the weights w_{ij} are calculated.

Using the system equation (8) we define further a local energy function between the pair of platforms p_i and p_j to be minimized

$$\begin{aligned} \tilde{J}_{ij} &= v_{d_i}^T v_{d_i} \\ &= a_{ij} + b_{ij}w_{ij} + c_{ij}(w_{ij})^2 \rightarrow \min \end{aligned} \quad (12)$$

where $\tilde{J}_{ij} \geq 0, a_{ij}, c_{ij} > 0$.

The question is how to combine the local energy function (12) and the utility function (10), and how are the parameters in (10) to be chosen? An intuitive choice

$$\tilde{b}_{ij} = |b_{ij}|, \quad \tilde{c}_{ij} = c_{ij} \quad (13)$$

guarantees $w_{ij} \geq 0$. It can also be shown that, independently of a_{ij} , near the equilibrium $v_{d_i} = 0$, and for $p_i = 1$, the energy function (12) reaches its minimum, and the utility function (10) its maximum, respectively.

With (13) the utility function (10) becomes

$$U_{ij} = |b_{ij}|w_{ij_c} - c_{ij}p_i(w_{ij_c})^2 \quad (14)$$

A maximization of (14) leads to

$$\frac{\partial U_{ij}}{\partial w_{ij_c}} = |b_{ij}| - 2c_{ij}p_iw_{ij_c} = 0 \quad (15)$$

from which a local w_{ij_c} is obtained

$$w_{ij_c} = \frac{|b_{ij}|}{2c_{ij}} \cdot \frac{1}{p_i} \quad (16)$$

Maximization of the local profit function (11) yields

$$\frac{\partial \rho_{ij}}{\partial w_{ij_p}} = g_{ij}p_i - 2e_{ij}w_{ij_p} = 0 \quad (17)$$

from which a local w_{ij_p} is obtained

$$w_{ij_p} = \frac{p_i}{2\eta_{ij}} \quad \text{where} \quad \eta_{ij} = \frac{e_{ij}}{g_{ij}} \quad (18)$$

The requirement for an equilibrium between the sums of the "produced" w_{ij_p} and the "demanded" w_{ij_c} leads to the balance equation

$$\sum_{j=1}^m w_{ij_c} = \sum_{j=1}^m w_{ij_p} \quad (19)$$

Substituting (16) and (18) into (19) gives the prices p_i for the weights w_{ij_p}

$$p_i = \sqrt{\frac{\sum_{j=1}^m |b_{ij}|/c_{ij}}{\sum_{j=1}^m 1/\eta_{ij}}} \quad (20)$$

Substituting (20) into (16) yields the final weights w_{ij} to be implemented in each local system. Once the new weights w_{ij} are calculated, each of them has to be normalized with respect to $\sum_{j=1}^m w_{ij}$ which guarantees the above requirement $\sum_{j=1}^m w_{ij} = 1$.

V. MB OPTIMIZATION OF OBSTACLE AVOIDANCE

In the following the optimization of obstacle avoidance between moving platforms by MB methods will be addressed. Coming back to the equation of the system of mobile robots (8)

$$v_{d_i} = vo_i + \sum_{j=1, i \neq j}^m w_{ij}v_{ij_p} \quad (21)$$

where vo_i is a subset of the RHS of (5) - a combination of different velocities (tracking velocity, control terms, etc.). v_{ij_p} reflects the repulsive forces between platforms P_i and P_j . The *global* energy function (12) reads

$$\begin{aligned} \tilde{J}_i &= v_{d_i}^T v_{d_i} \\ &= vo_i^T vo_i + 2vo_i^T \sum_{j=1, i \neq j}^m w_{ij}v_{ij_p} \\ &+ \left(\sum_{j=1, i \neq j}^m w_{ij}v_{ij_p} \right)^T \left(\sum_{j=1, i \neq j}^m w_{ij}v_{ij_p} \right) \end{aligned} \quad (22)$$

The *local* energy function reflects only the energy of a pair of two interacting platforms P_i and P_j

$$\begin{aligned} \tilde{J}_{ij} &= v_{d_i}^T v_{d_i} \\ &= vo_i^T vo_i + \left(\sum_{k=1, k \neq i, j}^m w_{ik}v_{ik_p} \right)^T \left(\sum_{k=1, k \neq i, j}^m w_{ik}v_{ik_p} \right) \\ &+ 2 \sum_{k=1, k \neq i, j}^m w_{ik}vo_i^T v_{ik_p} \\ &+ 2w_{ij}(vo_i^T + \sum_{k=1, k \neq i, j}^m w_{ik}v_{ik_p}^T)v_{ij_p} \\ &+ w_{ij}^2(v_{ij_p}^T v_{ij_p}) \end{aligned} \quad (23)$$

Comparison of (23) and (12) yields

$$\begin{aligned}
a_{ij} &= v_{o_i}^T v_{o_i} + \left(\sum_{k=1, k \neq i, j}^m w_{ik} v_{ik_p} \right)^T \left(\sum_{k=1, k \neq i, j}^m w_{ik} v_{ik_p} \right) \\
&+ 2 \sum_{k=1, k \neq i, j}^m w_{ik} v_{o_i}^T v_{ik_p} \\
b_{ij} &= 2(v_{o_i}^T + \sum_{k=1, k \neq i, j}^m w_{ik} v_{ik_p}^T) v_{ij_p} \\
c_{ij} &= v_{ij_p}^T v_{ij_p}
\end{aligned} \tag{24}$$

VI. SIMULATION RESULTS

The following simulation results consider mainly the obstacle avoidance of the multi-robot system in a relatively small area. The sensor cone of a platform amounts to +/- 170 deg. Inside the cone a platform can see another platform within the range of 0-140 units. Platforms P_1 and P_3 are approaching head-on. At the same time platform P_2 crosses the course of P_1 and P_3 right before their avoidance manoeuver. If there were only platforms P_1 and P_3 involved the avoidance manoeuver would work without problems. According to the built-in traffic rules both platforms would move some steps to the right (seen from their local coordinate system) and keep heading to their target after their encounter. Platform P_2 works as a disturbance since both P_1 and P_3 react on the repulsive potential of P_2 which has an influence on their avoidance manoeuver. The result is a disturbed trajectory (see Fig. 5) characterized by drastic changes especially of the course of P_3 during the rendezvous situation. A collision between P_1 and P_3 cannot be excluded because of the crossing of the courses of P_1 and P_3 .

When we activate the MB optimization, we obtain a behavior that follows both the repulsive potential law for obstacle avoidance and the traffic rules during approaching head-on (see Fig. 6). There is no crossing of tracks between P_1 and P_3 anymore due to the MB optimization of the repulsive forces between platforms P_1, P_2 , and P_3 and a respective tuning of the weights w_{ij} . Figure 7 shows the resulting weights. We also notice that w_{12} and w_{13} , w_{21} and w_{23} , and w_{31} and w_{32} are pairwise mirror-inverted due to the condition $\sum_{j=1, i \neq j}^m w_{ij} = 1$ (see also eq. (8)).

In further simulations, which are not shown here, the platforms move on circles with different speed, similar diameters and centerpoints while avoiding other platforms and static obstacles on their tracks. To determine the smoothness of the trajectories, the averages of the curvatures along the trajectories of the platforms were calculated. It turned out that the use of MB optimization leads to a significant improvement of the smoothness of trajectories (see Figs. 8 and 9).

VII. CONCLUSIONS

Navigation while avoiding obstacles by mobile robots can be successfully done by using a mixture of principles like virtual potential fields, traffic rules, and control methods. It has also been shown that a 'deformation' of the central symmetry

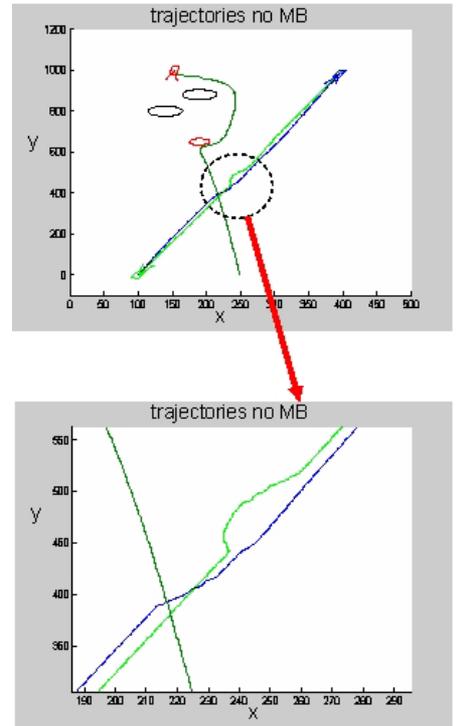


Fig. 5. Approach, no MB optimization

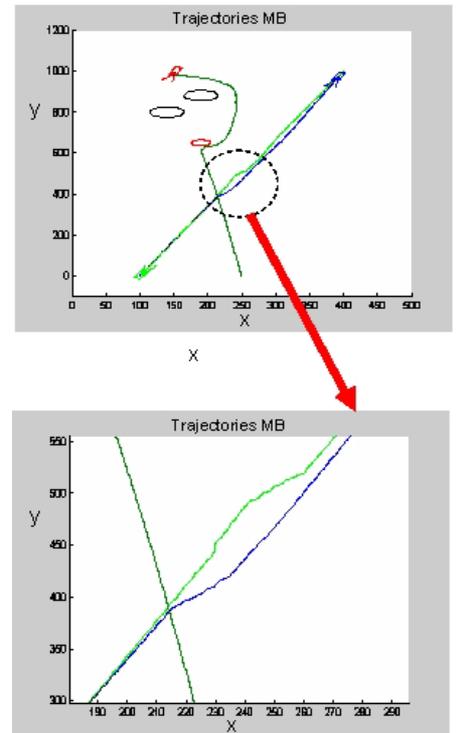


Fig. 6. Approach, with MB optimization

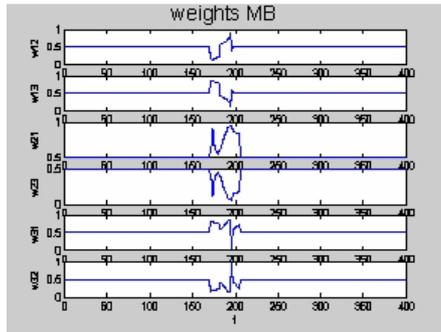


Fig. 7. weights, with MB

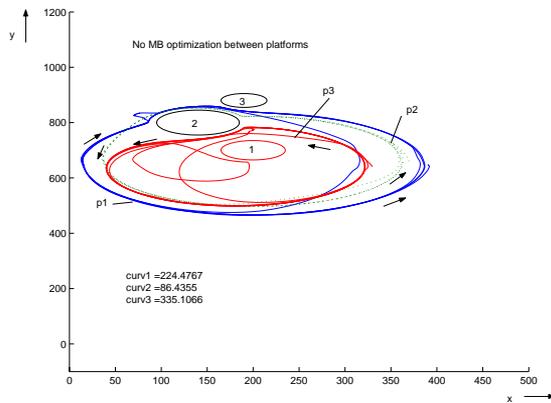


Fig. 8. moving on circular trajectories, no MB

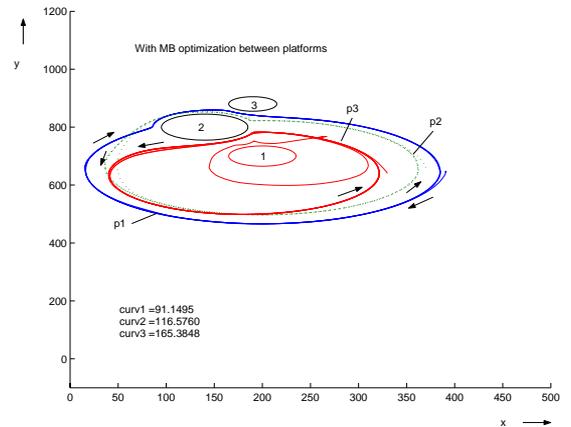


Fig. 9. moving on circular trajectories, with MB

of an obstacle is helpful because it takes into account whether a robot is moving towards or away from it. An important aspect presented in this paper is the market-based (MB) optimization of competing potential fields of mobile platforms. MB optimization imitates economical behavior between consumer and producer agents on the basis of a common price. By means of MB optimization some potential fields will be strengthened and some weakened depending on the actual scenario. This is especially required when more than two robots meet within a small area which makes a certain minimum distance between the robots and appropriate manoeuvres necessary. Therefore, MB navigation allows smooth motions in such situations. Simulation experiments with simplified robot kinematics and dynamics have shown the feasibility of the presented method. Although it does not belittle the presented results, it has to be underlined that the simulation of the vehicles only meet a part of the requirements for non-holonomic systems. Restrictions w.r.t. to abrupt changes of motions in position and orientation are only indirectly considered. Restrictions regarding the weight, height, engine and wheels of the vehicles and the sensor have been neglected to a large extend. Therefore, future aspects of this work are the enhancement of the simulation regarding more realistic vehicles with the goal of implementation on a real mobile robot.

REFERENCES

[1] O. Khatib. Real-time Obstacle avoidance for manipulators and mobile robots. *IEEE Int. Conf. On Robotics and Automation, St. Louis, Missouri, 1985*, page 500505, 1985.

- [2] Y. Koren and J. Borenstein. Potential field methods and their inherent limitations for mobile robot navigation. *Proceedings of the IEEE Conference on Robotics and Automation, Sacramento, California*, pages 1398–1404, April 7–12, 1991.
- [3] M. Aicardi and M. Baglietto. Decentralized supervisory control of a set of mobile robots. *Proceedings of the European Control Conference 2001.*, pages 557–561, 2001.
- [4] M. J. Sorensen. Feedback control of a class of nonholonomic hamiltonian systems. *Ph.D. Thesis, Department of Control Engineering Institute of Electronic Systems Aalborg University, Denmark.*, 2005.
- [5] J. Alonso-Mora, A. Breitenmoser, M. Ruffli, P. Beardsley, and R. Siegwart. Optimal reciprocal collision avoidance for multiple non-holonomic robots. *Proc. of the 10th Intern. Symp. on Distributed Autonomous Robotic Systems (DARS)*, Switzerland, Nov 2010.
- [6] A. Bouguerra. Robust execution of robot task-plans: A knowledge-based approach. *Ph.D. Thesis, Oerebro University*, 2008.
- [7] A. Wallace. Flow control of mobile robots using agents. *29th International Symposium on Robotics Birmingham, UK*, pages 273–276, 1998.
- [8] T.B. Gold, J.K. Archibald, and R.L. Frost. A utility approach to multi-agent coordination. *Proceedings of the 2000 IEEE International Conference on Robotics and Automation, San Francisco*, pages 2052–2057, 1996.
- [9] E.W. Large, H.I. Christensen, and R. Bajcsy. Scaling the dynamic approach to path planning and control: Competition among behavioral constraints. *Vol. 18, No. 1*, pages 37–58, 1999.
- [10] T. Teredesai and V.C. Ramesh. A multi-agent initiative system for real-time scheduling. *SMC98 Conference, San Diego, CA, USA*, pages 439–444, 1998.
- [11] M. B. Dias, R. Zlot, N. Kalra, and A. Stentz. Market-based multirobot coordination: a survey and analysis. *Proceedings of the IEEE, vol. 94, no. 7*, pages 1257–1270, July 2006.
- [12] S.H. Clearwater (ed.). Market-based control: A paradigm for distributed resource allocation. *Proceedings of the 38th CDC, Phoenix, Arizona USA.*, World Scientific, Singapore., 1996.
- [13] O. Guenther, T. Hogg, and B.A. Huberman. Controls for unstable structures. *Proceedings of the SPIE, San Diego, CA, USA*, pages 754–763, 1997.
- [14] H. Voos and L. Litz. A new approach for optimal control using market-based algorithms. *Proceedings of the European Control Conference ECC99, Karlsruhe*, 1999.
- [15] R. Palm. Synchronization of decentralized multiple-model systems by market-based optimization. *IEEE Trans Syst Man Cybern B, Vol. 34*, pages 665–72, Feb 2004.
- [16] B. R. Fajen and W. H. Warren. Behavioral dynamics of steering, obstacle avoidance, and route selection. *Journal of Experimental Psychology: Copyright by the American Psychological Association, Inc. Human Perception and Performance, Vol. 29, No. 2*, page 343362, 2003.
- [17] R. Palm, D. Driankov, and H. Hellendoorn. Model based fuzzy control. *Springer-Verlag Berlin New York Heidelberg*, 1997.