Maintaining Timelines with Hybrid Fuzzy Context Inference

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Abstract
Timelines allow to represent temporally-rich information about plans as well as the current execution status of plans. Recent work has addressed the related issue of inferring timelines representing contextual information — often useful for informing planning and/or plan execution monitoring processes. The present article addresses the particular issue of inferring context from given models of how observations relate to context, and representing this context on timelines. We strive to abandon assumptions currently made on context recognition, namely that hypotheses are either confirmed or disproved. We propose a technique which allows to accept the inferred context on a timeline with a degree of possibility. The approach is based on fuzzy constraint reasoning, and captures two sources of uncertainty: uncertainty in the model that is used to infer context, and uncertainty in the observations. We also formulate the problem of searching for the most likely timeline as a Constraint Optimization Problem.

Introduction
The concept of timeline is central to many planning and scheduling approaches. Timelines allow to represent temporally-rich information about plans as well as the current execution status of plans. Recently, several continuous planning approaches have been proposed which use timelines to maintain the on-going state of the domain as it is observed by sensors. This allows the planner to infer courses of action which are contextual to the current status of the world. Applications in which the temporal context is used in a closed loop with planning include domestic activity management (Pecora et al., 2012) and unmanned aerial vehicles (Doherty, Kvarnström, and Heintz, 2010). In these works, plan generation and execution depends in non-trivial ways on the temporal relationships among observations. These relationships are modeled in languages based on temporal constraints, and the algorithms which allow to infer the timelines representing context are based on temporal constraint propagation techniques. Examples of these algorithms include chronicle recognition (Dousson and Maigat, 2007) and abductive temporal inference (Pecora and Cirillo, 2009). The present article addresses this particular issue, namely inferring context from given models of how observations relate to context, and representing this context on timelines.

Timeline generation and maintenance can take into account some degree of uncertainty in the temporal placement of values. This is typically achieved by means of temporal constraint reasoning techniques (e.g., Simple Temporal Problems, STP (Dechter, Meiri, and Pearl, 1991)) which are capable of maintaining temporal bounds on observations and inferred values on the timeline. However, current approaches to context inference lack the ability to account for two important sources of uncertainty, namely uncertainty in the model that is used to infer context, and uncertainty in the observations. To motivate why this is useful let us begin with an example. Suppose we have the ability to observe the location of a person at home, as well as the current state of the kitchen stove. We wish to recognize the occurrence of the observed person being in the context of “Cooking”. We employ for this purpose a model which asserts that the user is cooking if the stove is on while he/she is in the kitchen. Now, suppose the scenario unfolds as follows: the user enters the kitchen and turns on the stove; after a while, he turns off the stove before leaving the kitchen. The stove state sensor is based on temperature: when the temperature goes below a certain threshold, the stove is considered to be off. Therefore, the sensor observations associated to the stove will indicate that it is still on after the user has left the kitchen. This situation falls outside the model stated above. The failure to recognize the cooking activity is not due only to sensor imprecision; it is also a consequence of the fact that model cannot anticipate all the possible states which can occur in a real situation. In other words, this failure is caused by the incapability of the inference process to deal with uncertainty in the model. Similarly, we may want to take into account the fact that the sensor providing the location observations gives uncertain readings. It may be the case, for instance, that its image processing algorithm returns an uncertain estimate of the person being in the kitchen, and that we want to factor this information into the context recognition process in order to provide a degree of belief of the cooking situation.

Two strategies can be adopted in order to extend the ap-
plicability of a model under uncertainty. First, to replace the modeling language with one that can explicitly include measures of uncertainty; second, to measure the applicability of a crisp model to the current situation using an underlying theory of uncertainty. In this work, we follow the latter approach. By doing so, we maintain the ability to use a simple model to represent domain knowledge, and we accommodate uncertainty by modifying the inference process. We use techniques based on fuzzy logic to measure the similarity between the situation reported by the sensor readings and the situation described in the model. We leverage fuzzy constraint-based reasoning in an abductive reasoning process to recognize a context on the basis of a set of heterogeneous sensor readings, which may contain uncertainty. Note that we are not interested in how the uncertainty in the sensor readings is computed: we assume that sensors come with an appropriate sensor model. The use of fuzzy temporal inference techniques may result in general in the generation of multiple timelines which are compatible, to some degree, to the sensor data. Accordingly, in the last part of this paper we present an approach to extract a unique, most likely timeline.

This paper is organized as follows. First, the overall context recognition algorithm is explained. This algorithm relies on two constraint-based inference processes, namely propagation in a fuzzy value constraint network and in a fuzzy temporal constraint network. After detailing these constraint reasoning procedures, we investigate two important problems caused by accommodating uncertainty in the inference process, namely (1) that of determining quantified temporal bounds for the inferred context, and (2) that of determining the most likely timeline.

**Constraint-based Context Modeling**

In this work, we leverage a context inference algorithm similar to the one described by Pecora et al. (2012). A context is an inferred value (e.g., activity of a human) which is the result of inference based on the observable properties in an environment (e.g., sensor readings). Both observable properties and inferred values are represented as state variables. For instance, a state variable Human can represent the current activity of a user, e.g., {Cooking, Eating, Relaxing, Sleeping}, or model the possible values of observable properties of the environment. We refer to these values as sensor readings e.g., a state variable can represent a Stove whose values can be {On, Off}.

Correlations among the values of state variables are defined in a model in terms of two dimensions of knowledge. One is the value dimension, which refers to the possible values a state variable can assume (e.g., Human = Cooking). The other dimension is the temporal dimension, which is formulated as relations in Allen’s Interval Algebra (Allen, 1984). These relations are qualitative, and model the relative placement in time of state variables values (e.g., Human = Cooking During Stove = On). In this example, a “=” relation is imposed on the values of both the Human and Stove state variables. Note that similar approaches (Pecora et al., 2012; Dousson and Maigat, 2007) implicitly assume an “=” relation on values. In our approach, we admit the “̸=” relation as well, e.g., it is possible to assert in the model that Cooking depends on Location ≠ Bedroom.

The collection of temporal and value relations constitutes the model based on which context inference occurs. Relations which assert the same value on a state variable that represents an inferred property of the environment (e.g., Human) are collected into so-called rules. These rules are from a domain model. For instance, the following rule describes one possible condition under which the human activity of Cooking can be inferred:

\[(\text{Human} = \text{Cooking}) \land (\text{Location} = \text{Kitchen}) \land (\text{Human} = \text{Cooking}) \land (\text{Stove} = \text{On})\]

Given a rule like the one above, we call head of the rule, the value of the state variable representing the inferred property (e.g., Human = Cooking), and we call requirements the relations and values of the other state variables involved in the rule.

The entire inference process is an abductive reasoning process, whereby observed or previously inferred values are explained by hypothesizing the occurrence of specific values and testing these hypotheses repeatedly by using the rules in the model. Abductive reasoning imposes the requirements of a rule as constraints between an interval \(h\), which represents a hypothesis of the head of the rule occurring, and other intervals representing sensor readings as they have been observed. Intervals representing sensor readings are maintained in a so-called Sensor Constraint Network (SCN, Figure 1). The SCN contains all sensor readings and the relations among them as they are observed. The resulting set of constraints is propagated in order to decide whether the hypothesis is admissible.

**Context inference with uncertainty**

The context inference is done through the abductive reasoning process on patterns of sensor observations and places these patterns and temporal relations together with the model in a constraint network. Temporal constraint propagation ascertains whether what is hypothesized has occurred. However, in current approaches inference has a Boolean result: either the hypothesis is confirmed or it is disproved. These approaches would fail to recognize an activity like cooking in a real world scenario (see the example in Introduction). As mentioned, the lack of flexibility in the model is what caused the failure. One way to tackle this problem is to have a much larger set of rules that captures many occurrences of sensor patterns. This approach has important disadvantages. One is that it is cumbersome to model a collection of rules that are able to anticipate all the possible relative temporal relations among the sensor reading patterns. More importantly, increasing the number of rules enlarges the search space in an abductive reasoning process in which each single rule should be hypothesized and propagated within a constraint network. In our approach, we follow a different strategy to have a conservative extension of the crisp case, namely we employ uncertain inference to relax the model. Specifically, we employ the notion of fuzzy constraint satisfaction problem (Dubois, Fargier, and Prade, 1996). We replace crisp temporal constraints with fuzzy temporal constraints, therefore, computing a possibility de-
Inference in the fuzzy value and temporal networks is explained in the following sections. Note that once an overall possibility degree for a hypothesis is obtained, we are left with the task of deciding temporal bounds for the inferred hypothesis. This requires the use of quantitative temporal constraints, which we propagate in a STP to obtain the bounds of inferred hypotheses according to their most likely temporal placement. This latter procedure yields timelines, each of which has an associated possibility degree. The problem of finding the timeline which has the maximum possibility degree, therefore the timeline representing the most likely overall sequence of inferred context, can be cast as a constraint optimization problem. This last aspect of our work is currently being investigated, and therefore we limit the description of the search of the most likely timeline to problem definition.

**Fuzzy Value Constraint Network**

A fuzzy constraint network (fuzzy CN) is a triple \( \langle X, D, C \rangle \) where \( X \) and \( D \) are a finite set of variables and their domains, and \( C \) is a set of fuzzy constraints. The fuzzy value constraint network which is built for the purpose of this work includes two categories of variables. One consists of the variables which model the value requirements of a hypothesis, and the other represents sensor readings. The domains \( D \) of all variables are the symbols representing possible states (e.g., \{On, Off\} for the variable representing the Stove). A fuzzy constraint is a fuzzy relation \( R \) on a set of variables \( V \subset X \) which is denoted as \( R_V \). This relation, that is, a fuzzy set of tuples, is defined by a membership function \( \mu_{R_V} \) (Klir and Folger, 1988). Each tuple \( t_V \in R_V \) is an assignment of values to the variables in \( V \), and the membership function assigns a degree of possibility in \([0, 1]\) to each tuple. Notice that possibility degrees do not need to add to one: for example, the void constraint over the values of \( V \) is represented by \( \mu_{R_V}(t_V) = 1 \) for all \( t_V \). The projection of a tuple \( t \) over a sequence of variables \( V \) is denoted by \( t[V] \).

A solution of a fuzzy CN is a complete assignment for all variables in \( X \) with satisfaction degree greater than 0. The satisfaction degree of a complete assignment \( t \) is

\[
\text{deg}(t) = \min_{R_V \in C} \mu_{R_V}(t[V])
\]

The optimal solution \( \hat{t} \) of a fuzzy CN is the complete assignment whose membership degree is maximum over all complete assignments (Ruttkay, 1994), that is,

\[
\hat{t} = \arg \max_{t \in \prod_{i \in X} D_i} \text{deg}(t)
\]

Sensor processes continuously add to the SCN new variables which represent perceived sensor values. The degree of belief of a particular value is modeled as a soft unary constraint. For example, the belief that the stove is on with possibility \( \alpha_1 \) and off with possibility \( \alpha_2 \) is modeled with a unary constraint whose membership function imposes \( \alpha_1 \) and \( \alpha_2 \) on the values On and Off, respectively. A possibility degree of 0 or 1 is also assigned to each value of the state variables which model the value requirement. As shown in Figure 2 for instance, the variable “Stove2”, representing the...
perceived state of the stove, is constrained to assume values On = 0.4 and Off = 1.0. Another variable “Stove1” is used to represent the value requirement of a rule asserting that the stove should be on. If \( X = \{ x_1, ..., x_k \} \) is a finite set of \( k \) variables, the membership function for the unary fuzzy constraints imposed on every variable \( x_i \in X \) is expressed as

\[
\mu_{R_e}(t) \to [0, 1] \quad t \in D_i
\]

Binary constrains of the fuzzy value constraint network are hard constraints. This is because binary constraints represent the equality and inequality requirements of a rule (e.g., we want the Stove = On). For example, a tuple \((a, a)\) which is a possible assignment for two variables, is assigned to the value 1 in case of an “=” constraint, and to value 0 for the case of the inequality constraint. The membership function for a binary hard constraint over variables \( w_{ij} = \{ x_i, x_j \neq i \} \) is defined as

\[
\mu_{R_{w_{ij}}}(t) \to \{0, 1\} \quad t \in D_i \times D_j
\]

In the example shown in Figure 2, a binary constraint is used to enforce that the value requirement “Stove1” is equal to the perceived state represented by variable “Stove2”.

In order to obtain a maximum possibility degree of the fuzzy constraint network, arc consistency and search are performed. Since the structure of the network in this specific application is a tree, the problem of finding the maximum satisfaction degree can be solved in polynomial time (Dechter, 2003). In fact, the structure of the network follows the semantics of unification, in which there is a value relation (= or \( \neq \)) obtained from a rule between the variable modeling the value requirement and the variable modeling a sensor reading, therefore, the structure is always a tree.

The maximum possibility degree of the network shown in Figure 2 is 0.4. If the binary constraint stated in the rule had been an inequality, the value possibility of the network would have been 1.0. The semantics of inequality are omitted here and explained in more detail by Mansouri (2011).

**Figure 2:** A fuzzy CSP with two state variables, three constraints (Soft unary constraint \( \{c_1, c_3\} \), Hard binary constraint \( c_2 \)) and domain \( \{\text{On, Off}\} \). 0.4 for On and 1.0 for Off are data obtained from sensor

During the execution, the meaning of the constraint is modified (Mansouri, 2011). The semantics of inequality are omitted here and explained in more detail by Mansouri (2011).

**Figure 3:** The Allen relations and their membership grades with respect to the relation Equals. In this figure, Allen’s relations are defined as follows: Meets(m), During(d), Before(<), Overlaps(o), FinishedBy(f), Contains(dt), StartedBy(s), Equals(=), Starts(s), Finishes(f), OverlappedBy(o), MetBy(m), After(>). (Adapted from Guesgen (2002).)

**Fuzzy Temporal Constraint Network**

In the previous section, we addressed how to check the value eligibility. In addition to the value constraints, we have to consider the temporal requirements in the rules. As explained in the section “Constraint-based Context Modeling”, the temporal constraint network is created by imposing temporal requirements of the hypothesis on SCN. We call this network, “original temporal constraint network”. The objective is to find the consistent temporal constraint network that is closest to the original one. If the original temporal constrains network is not consistent, we find the closest consistent temporal constraint network through introducing uncertainty to the original network. The notion of flexibility and uncertainty for the temporal aspect of the model are provided by fuzzifying Allen relations (Guesgen, 2002). A fuzzy Allen relation is represented as a set of crisp Allen relations with an associated possibility degree. To define the membership grade, the notion of conceptual neighborhood is leveraged (Freksa, 1992). For instance, assume that two intervals \( I_1 \) and \( I_2 \), are in relation Equals, then by allowing the duration of the intervals to vary, we can change this relation to During or Contains. In this case, to make a fuzzy set including a pair of all thirteen Allen relations, we assign the membership grade 1 to the relation Equals and a membership grade less than 1 for the others. The membership grades are defined for each relation based on the closeness to the Equals relation. Figure 3 illustrates this example in the topological view of conceptual neighborhood with the membership grades, \( 1 = \alpha_0 \geq \alpha_1 \geq \alpha_2 \geq ... \geq 0 \)

We divide Allen relations in the temporal constraint network into two categories. The first category belongs to the relations imposed from a rule and the second is for the relations capturing the relative position in time of the sensor readings. Since we want to determine the degree of possibility of a rule in the model given existing sensory patterns, we fuzzify the relations of the first category in the way ex-
plained earlier. As for the second category, Allen relations remain as crisp constraints. In other words, for Allen relations that are responsible for placing sensor readings in time, the initial relation is assigned to 1, while the membership grade of the others is 0.

In order to have an optimal solution for the fuzzy Allen network (i.e., a crisp, consistent temporal constraint network that is closest to the original temporal constraint network), we use a generalization of the classical backtracking algorithm with incremental path consistency to the fuzzy framework (Badaloni and Giacomini, 2000). More precisely, by applying a fuzzy extension of Allen’s algorithm, we obtain the maximum temporal possibility degree as well as the sets of complete assignments with the highest possibility degree. The maximum temporal possibility is calculated by taking the minimum over the maximum membership degree of each edge in the propagated fuzzy temporal constraint network.

In crisp temporal constraint reasoning, finding a feasible solution to the interval constraint problem is solved with a combination of constraint propagation (path consistency) and search. The complexity of the fuzzy extension to Allen’s propagation algorithm is augmented at most by a factor equal to the number of levels of preferences used to define the fuzzy Allen network (Badaloni and Giacomini, 2000). In our work, there are as many levels of preference as there are conceptual neighbors.

An example of fuzzy temporal reasoning is shown in Figure 4. With respect to Figure 4(a), the relation During* is fuzzified as follows: \{ (Before, 0.2), (Meets, 0.4), (Overlaps, 0.6), (FinishedBy, 0.4), (Contains, 0.6), (StartedBy, 0.4), (Equals, 0.8), (Starts, 0.8), (During, 1), (Finishes, 0.8), (OverlappedBy, 0.6), (MetBy, 0.4), (After, 0.2) \} and the relation During** is a crisp relation with the membership degree of 1 for During and 0 for the rest of Allen’s relations. By applying propagation on the fuzzy temporal network depicted in Figure 4(a), these membership degrees are updated. The optimal solution is built through the search process. In this example, we obtain the maximum possibility degree of 1 as a result of taking minimum over membership degrees of the relations in the optimal solution. Having maximum possibility degree of 1 indicates that this network is temporally consistent. Clearly, in this case, the closest temporal constraint network is the original one.

Suppose the case that the Allen relation During** which models the temporal relation of sensor reading patterns is replaced by an OverlappedBy relation. This case is compatible with the Cooking scenario explained in Introduction Section, in which the network was not temporally consistent (see Figure 4(b)). By fuzzifying this network and propagating the resulting network, we obtain a maximum possibility degree of 0.6. Several selections of Allen’s relations entail this possibility degree. One of them is represented in Figure 4(c). In other words, the network shown in Figure 4(c) represents the “closest interpretation” of the temporal relations among sensor readings that would make the rule consistent. The “similarity” between this interpretation and the one required by the rule is 0.6.
Determining the Overall Possibility Degree

We explained two fuzzy constraint networks which ascertain temporal satisfiability and value satisfiability. Now, to obtain an overall possibility degree of an inferred value from the two individual degrees, we are facing to the problem of multi criteria aggregation. Different operators can be employed to achieve this aggregation. A primary factor in the determination of the structure of such aggregation is the relationship between the criteria involved. Consider the cases of wanting, for example, “all” or “at least one of them” or “most” of the criteria to be satisfied. For each of these cases and many of which are not contemplated here, a specific operator is proposed, e.g., t-norm, t-conorm and ordered weighted averaging (OWA) operators (Yager, 1988). In our problem, we desire that both evaluating criteria be satisfied; so, a t-norm operator can be an appropriate choice. In this work, we employ the minimum possibility degree of the two constraint networks as the overall degree. One might also wish to keep the two possibility degrees separate, to be able to query on the degree of satisfiability for each dimension.

Computing Interval Bounds of Inferred Value

The propagation of qualitative Allen relations does not provide any knowledge about the time bounds of the intervals. I.e., we know the temporal and value possibility of an inferred activity occurring, but we do not know when it could occur, except for a relative qualitative placement in time of sensor readings. In fact, we know when sensor readings occur in time, but the lack of knowledge about the occurrence interval of inferred values makes it impossible to build a timeline. Since a timeline corresponding to a state variable is a sequence of values in time, the need of quantitative temporal reasoning arises; thus, we convert the obtained optimal temporal network containing a solution with maximum possibility degree to a STP. As noted earlier, the optimal solution contains a temporally consistent network with the highest possibility degree relative to the initial network, thus, the network which is converted to STP is consistent. Temporal propagation in the STP is done through the Floyd-Warshall algorithm, whose computational cost is \(O(n^3)\). Therefore, computing the temporal bounds for the inferred value can be done in polynomial time.

As a case in point, the network shown in Figure 4(c) is converted to a STP. This network is consistent as a result of performing the fuzzy Allen algorithm on the inconsistent network shown in Figure 4(b). Since the \(\text{Stove} \) and \(\text{Location} \) state variables represent sensor readings, we know the precise time in which these readings occurred. For instance, if the user was in the kitchen in the interval \([1, 12]\) and the stove was on in the interval \([5, 15]\), then the occurrence of Cooking would be \([0, 16]\). The cooking interval is calculated by choosing the earliest time of the timepoints in the STP which represent the beginning and ending of the Cooking activity.

Dealing with Multiple Timelines

The abductive reasoning process, by hypothesizing the head of rules, outputs hypotheses which can have different possibility degrees. The number of hypotheses is the number of combinations of state variable values in the SCN which can be unified with the value requirements of the applied rule. For instance, in the case that Cooking is hypothesized which is shown in the Figure 4, there are six possible combinations of state variables \(\text{Stove} \) and \(\text{Location} \). Each combination entails an inferred hypothesis with a particular possibility degree. Moreover, there can be dependencies among the rules, therefore, each inferred value can be a requirement for the others. For instance, the model may state that Eating occurs after Cooking (among other requirements), thus, inferring the occurrence of Eating depends on a Cooking hypothesis which has been already inferred. Consequently, each Cooking hypothesis enlarges the number of combinations of values which is needed for the process of inferring Eating. With respect to the growing rate of inferences, we aim at recognizing a most likely timeline. The possibility of a timeline is expressed as the minimum of the possibility degree assigned to the values of state variables in the model.

We cast the problem of extracting the best timeline (i.e., the timeline with maximum possibility degree) as a Constraint Optimization Problem, COP (Dechter, 2003), in which variables are hypotheses referring to the possible values of a state variable: values of this constraint network are different ways to “support” a value of a state variable which itself is a constraint network (combinations of state variable values in the SCN). Constraints are the value constraints and the Allen temporal relations among values of state variables prescribed in the rules. Assigning a support to the variable has a cost which is calculated through the fuzzy inference process. An optimal solution to this problem, is a set of supports assigned to the variables and has the highest possibility degree with regard to the constraints. The collection of intervals associated to the hypotheses belonging to the optimal solution is the most likely timeline of a state variable.

Example

To extend our Cooking scenario, the human user leaves the kitchen and goes to the dining room in order to eat his lunch. The inference system aims at recognizing both Cooking and Eating activities. The value and temporal requirements of Eating are modeled in the rule shown below

\[
(\text{Human} = \text{Eating}) \land (\text{Location} = \text{Dining table}) \land (\text{Human} = \text{Eating}) \land (\text{Human} = \text{Cooking})
\]

An example of sensing process is depicted in the Figure 5 which is compatible with the above scenario. In this Figure, in addition to the timelines of state variables \(\text{Location} \) and \(\text{Stove} \), we show the most likely timeline for state variable \(\text{Human} \) resulting from the overall process described in this paper.

The best two timelines of Human activity in terms of the possibility degrees are shown in Figure 6. The possibility degree associated to each value is the result of the fuzzy inference process which is introduced in this paper.
We have presented a fuzzy inference process for interpreting a crisp constraint-based model so as to accommodate uncertainty in sensor data and imprecision in the model. This fuzzy inference process covers two aspects of the model, namely value and temporal requirements. The problem of accommodating uncertainty into each aspect is solved using the notion of soft unary constraints in a fuzzy value constraint network and conceptual neighborhoods in a fuzzy temporal constraint network. We leverage state of the art algorithms for fuzzy qualitative temporal reasoning, and introduce quantified temporal bounds for the purpose of extracting timelines as a polynomial time post-processing step.

Our approach has been implemented in a system, which has been used on simple artificial scenarios like the ones shown in this paper. Our current work focuses on the evaluation of the system along two axes: first, time performance in larger domains; second, behavior in a real environment with sensors deployed in a smart home (Saffiotti et al., 2008).

While we have focused on qualitative temporal relations in this paper, it might also be interesting to investigate the use of quantitative temporal relations with uncertainty (Vidal and Fargier, 1999) and preferences (Khatib et al., 2001). Furthermore, it is essential to combine multiple hypotheses into candidate timelines. As described in this paper, we cast the problem of extracting a unique timeline as a COP, hence, different approaches for solving a COP which can be employed in this work should be studied.

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