Market-Based Algorithms and Fuzzy Methods for the Navigation of Mobile Robots

Rainer Palm, *Senior Member IEEE*, and Abdelbaki Bouguerra, *Member IEEE*

***Abstract*—An important aspect of the navigation of mobile robots is the avoidance of static and dynamic obstacles. This paper deals with obstacle avoidance using artificial potential fields and selected traffic rules. The potential field method is optimized by a mixture of fuzzy methods and market-based optimization (MBO) between competing potential fields of mobile robots. Here, depending on the local situation, some potential fields are strengthened and some are weakened. The optimization takes place especially when several mobile robots act in a small area. In addition, to avoid an undesired behavior of the mobile platform in the vicinity of obstacles, central symmetrical potential fields are 'deformed' by using fuzzy rules.**

I. INTRODUCTION

In the last two decades several methods of robot navigation and obstacle avoidance have been discussed. One of the most prominent methods for obstacle avoidance is the artificial potential field method (see [1]). Borenstein and Koren gave a review on this method addressing its advantages and disadvantages with respect to stability and deadlocks (see [2]). Another approach can be found in ([3]) where local groups of robots share information on common potential field regions for navigation among static and dynamic obstacles. Further research results regarding navigation of non-holonomic mobile robots can be found in [4] and [5]. The execution of robot tasks based on semantic domain-knowledge has been reported in detail by [6].

These few examples show the variety of methods for performing different subtasks like

- reaching a target
- avoiding obstacles
- following traffic rules

under the assumption of stable trajectories. A most successful method to cope with obstacle avoidance is the *fuzzy logic approach* which has been widely used for mobile robots since the early ninetieth. Martinez et al described a system of heuristic rules based on interaction of mobile robots and traffic rules [7]. A fuzzy obstacle controller using so-called negative-fuzzy rules is reported by Lilly [8] where a negative rule is a rule like "IF A THEN DO NOT B" in contrast to a positive rule "IF A THEN DO B". Stingu and Lewis combined a motion control fuzzy rule base using an occupancy map of the environment similar to an artificial potential field within which the robots interact [9].

Trying to achieve different tasks at the same time makes a decentralized optimization necessary, which generates different weights for the tasks. Decentralized methods like multi-agent control can handle optimization tasks for a large number of complex local systems more efficiently than centralized approaches. Mobile robot navigation is a important application for agent based control. One popular example is the flow control of mobile platforms in a manufacturing plant using intelligent agents (see [10]). One of the most interesting and promising approaches to cope with large decentralized systems is the market-based optimization (MBO). MB algorithms imitate economical systems where producer and consumer agents both compete and cooperate on a market of commodities.

[11] give an overview on MB multi-robot coordination, which is based on bidding processes. The method deals with motion planning, task allocation and team cooperation, whereas obstacles are not considered. [12] describe a MB recource allocation method for vehicle routing applications. This method is based on auction mechanisms where the trucks and the auctioneer are modeled as local agents with planning and bidding capabilities.

In order to improve the performance of safe navigation of multiple robots based on artificial potential fields the present paper adopts many ideas from [7], [13], [14], [15], [16], and [17] in order to combine fuzzy methods and MBO methods.

In the context of MB navigation, combinations of competing tasks, that should be optimized, can be manifold, for example the presence of a traffic rule and the necessity for avoiding an obstacle at the same time. Another case is the accidental meeting of more than two robots within a small area. This requires a certain minimum distance between the robots and appropriate (smooth) maneuvers to keep stability of trajectories to be tracked. This paper addresses exactly this point where optimization takes place between "competing" potential fields of mobile robots: Some potential fields are strengthened and some are weakened by a combination of MBO and fuzzy methods depending on the local situation. Repulsive forces both between robots and between robots and obstacles are computed under the assumption of central symmetrical force fields meaning that forces are computed between the centers of mass of the objects considered.

Section II addresses the navigation principles applied to the task. In Section III navigation and obstacle avoidance using potential fields and fuzzy rules in the framework of a multi-robot system is outlined. Section IV gives an introduction to the MB optimization used in this paper. The connection between the MB approach and the system to be controlled is outlined in Section V. Section VI shows simulation experiments and Section VII draws conclusions and highlights future work.

II. Navigation principles

A multi-robot system is constituted of individual mobile robots whose functions can be arranged with the help of a control hierarchy architecture which adopts the idea of a control hierarchy for industrial robots introduced by [18].

The navigation of a mobile robot is more or less located in the control levels ”High level control” and ”Trajectory Planner” receiving information from higher and lower control levels, and from the environment that consists of targets, obstacles, moving objects (e.g. other robots), and possible team members. To illustrate the navigation problems, let n mobile platforms (autonomous mobile robots) perform special tasks in a working area like loading materials from a starting station, bringing them to a target station and unloading the materials there. The task of the platforms is to reach their targets while avoiding obstacles and other platforms.

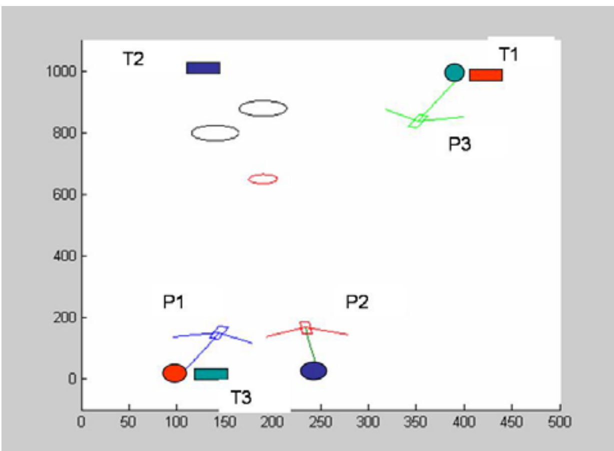

Fig. 1. Platform area

Navigation principles for a mobile robot (platform) P_i are meant to be *heuristic rules* to perform a specific task under certain restrictions originating from the environment, obstacles O_j, and other robots P_j. As already pointed out, each platform P_i is supposed to have an estimation about position/orientation of itself and the target T_i. The position of another platform P_j relative to P_i can be measured if it lies within the sensor cone of P_i. Four navigation principles are used here

1. Move in direction of target T_i

2. Avoid an obstacle O_j (static or dynamic) if it appears in the sensor cone at a certain distance. Always orient platform in direction of motion

3. Decrease speed if dynamic (moving) obstacle O_j comes from the right

4. Move to the right if the obstacle angles β (see [19]) of two approaching platforms are small
(e.g. $\beta < 10$) (see Fig. 2)

Let, for example, mobile robots (platforms) P_1, P_2, and P_3 be supposed to move to targets T_1, T_2, and T_3, respectively, whereas collisions should be avoided (see Fig. 1).

Apart from the *heading-to-target movement* all other navigation calculations and actions take place in the local coordinate system of platform P_i. The positions of obstacles (static or dynamic) O_j or other platforms P_j are also formulated in the local frame of platform P_i.

III. Navigation and obstacle avoidance using potential fields

A. Modeling of the system

The kinematic of the non-holonomic vehicle is described by

$$\begin{aligned}
\dot{q}_i &= R_i(q_i) \cdot u_i \\
q_i &= (x_i, y_i, \theta_i, \phi_i)^T \\
R_i(q_i) &= \begin{pmatrix} \cos\theta_i & 0 \\ \sin\theta_i & 0 \\ \frac{1}{l_i}\cdot\tan\phi_i & 0 \\ 0 & 1 \end{pmatrix}
\end{aligned} \tag{1}$$

where
$q_i \in \Re^4$ - state vector
$u_i = (u_{1i}, u_{2i})^T \in \Re^2$ - control vector, pushing/steering force
$x_{ip} = (x_i, y_i)^T \in \Re^2$ - position vector of platform P_i
θ_i - orientation angle
ϕ_i - steering angle
l_i - length of vehicle
Subscript d denotes the desired variable.

The tracking velocity is designed as a control term

$$v_{ti} = k_{ti}(x_{ip} - x_{ti}) \tag{2}$$

$x_{ti} \in \Re^2$ - position vector of target T_i
$k_{ti} \in \Re^{2\times 2}$ - gain matrix (diagonal)

Repulsive forces exist between platform P_i and obstacle O_j leading to repulsive velocities

$$v_{ij_{ob}} = -c_{ij_{ob}}(x_{j_p} - x_{j_{ob}})d_{ij_{ob}}^{-2} \tag{3}$$

$v_{ij_{ob}} \in \Re^2$ - repulsive velocity vector between platform P_i and obstacle O_j
$x_{j_{ob}} \in \Re^2$ - position vector of obstacle O_j
$d_{ij_{ob}} \in \Re$ - Euklidian distance between platform P_i and obstacle O_j
$c_{ij_{ob}} \in \Re^{2\times 2}$ - gain matrix (diagonal)

Repulsive forces also appear between platforms P_i and P_j from which we get the repulsive velocities

$$v_{ij_p} = -c_{ij_p}(x_{ip} - x_{j_p})d_{ij_p}^{-2} \tag{4}$$

$v_{ij_p} \in \Re^2$ - repulsive velocity between platforms P_i and P_j
$d_{ij_p} \in \Re$ - Euclidian distance between platforms P_i and P_j
$c_{ij_p} \in \Re^{2\times 2}$ - gain matrix (diagonal)

The resulting velocity v_{di} is the sum

$$v_{di} = v_{ti} + \sum_{j=1}^{m_{ob}} v_{ij_{ob}} + \sum_{j=1}^{m_p} v_{ij_p} \tag{5}$$

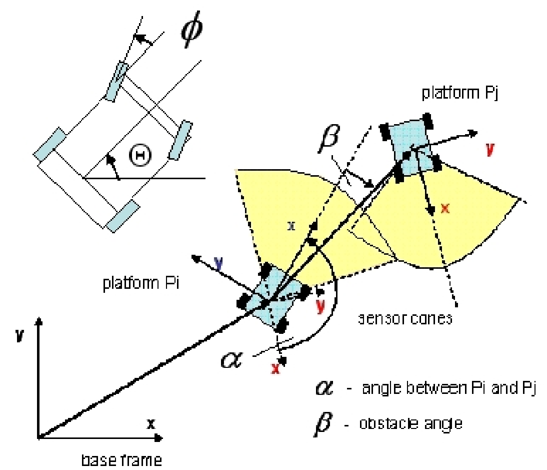

Fig. 2. Geometrical relationship between platforms

where m_{ob} and m_p are the numbers of contributing obstacles and platforms. It should be emphasized that the force fields are switched on/off according to the actual scenario: distance between interacting systems, state of activation according to the sensor cones of the platforms, positions and velocities of platforms w.r.t. to targets, obstacles and other platforms. All calculations of the velocity components (1)-(5), angles and sensor cones are formulated in the local coordinate systems of the platforms (see Fig. 2).

B. "Deformation" of potential fields using fuzzy rules

Potential fields of obstacles (static and dynamic) act normally independently of the attractive force of the target. This may cause unnecessary repelling forces especially in the case when the platform can "see" the target.

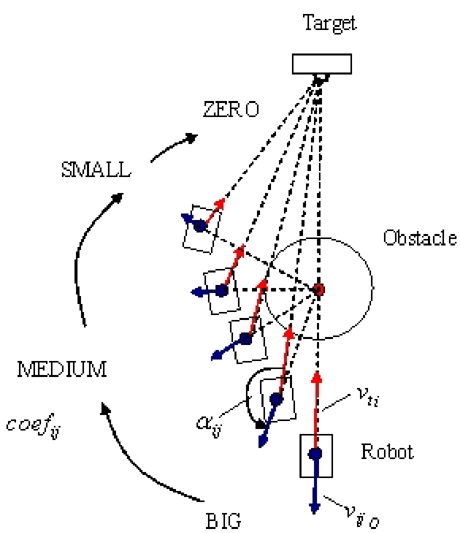

Fig. 3. Deformation of potential field

Another situation occurs when the tracking velocity $|v_{ti}|$ becomes zero for some reason. In this case a platform would be pushed away from an obstacle even if it should keep its position. The goal is therefore to "deform" the repulsive

v_{ti} \ α_{ij}	Z	S	M	B
B	Z	S	M	B
M	Z	S	M	M
S	Z	M	M	S
Z	Z	Z	Z	Z

Fig. 4. Fuzzy table for potential field

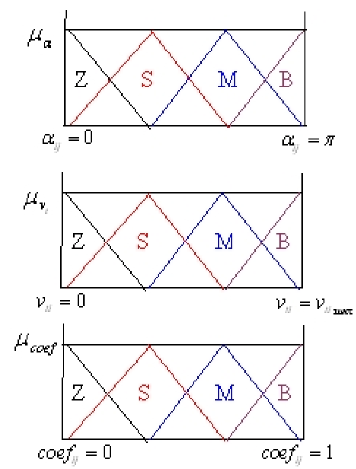

Fig. 5. Fuzzy membership functions

potential field so that it is strong if the obstacle hides the target and weak if the target "can be seen" from the platform. In addition, the potential field should also be strong for a high tracking velocity and weak for a small one (see Fig. 3). These requirements can be achieved by introducing a coefficient $coef_{ij} \in [0, 1]$ that is multiplied to $v_{ij\,ob}$ to obtain a new $v_{ij\,ob}$ as follows

$$v_{ij\,ob} = -coef_{ij} \cdot c_{ij\,ob} \cdot (x_{ip} - x_{j\,ob}) d_{ij\,ob}^{-2} \qquad (6)$$

The coefficients $coef_{ij}$ can be calculated by a set of 16 fuzzy rules like

$$\begin{aligned} IF \quad |v_{ti}| = B \quad AND \quad \alpha_{ij} = M \\ THEN \quad coef_{ij} = M \end{aligned} \qquad (7)$$

where α_{ij} is the angle between $v_{ij\,ob}$ and v_{ti}. The set of 16 rules can be summarized in a table shown in Fig. 4. Z - ZERO, S - SMALL, M - MEDIUM, B - BIG are fuzzy sets (see [20]). The corresponding membership functions μ_α, μ_{vt}, and μ_{coef} are triangular and shown in Fig. 5.

Finally can (5) be rewritten into

$$v_{di} = v_{ti} + \sum_{j=1}^{m_{ob}} w_{ij\,fuzz} v_{ij\,ob} + \sum_{j=1}^{m_p} v_{ij\,p} \qquad (8)$$

where

$$w_{ij\,fuzz} = coef_{ij} = \frac{\sum_{l=1}^{s} \mu^l(|v_{ti}|, \alpha_{ij}) \cdot coef_{ij}^l}{\sum_{l=1}^{s} \mu^l(|v_{ti}|, \alpha_{ij})} \quad (9)$$
$$\mu^l = min(\mu_{vt}, \mu_\alpha)$$

s - number of rules.

IV. MB APPROACH

The behavior of the multiple mobile robot system is optimized by an appropriate weighting of the repulsive forces/velocities $v_{ij\,ob}$ and $v_{ij\,p}$ using MBO methods. The desired motion of platform P_i is then described by

$$v_{d_i} = vo_i + \sum_{j=1, i\neq j}^{m_p} w_{ij} v_{ij\,p} + \sum_{j=1, i\neq j}^{m_{ob}} w_{ij\,ob} v_{ij\,ob} \quad (10)$$

where vo_i is a combination of

- tracking velocity depending on distance between platforms i and goals i
- Traffic rules

w_{ij} - weighting factors for repelling forces where $\sum_{j=1, i\neq j}^{m_p} w_{ij} = 1$
$w_{ij\,ob}$ - weighting factors for repelling forces between platform i and obstacle j.

The first objective is to change the weights w_{ij} so that all contributing platforms show a smooth dynamical behavior during avoiding each other. One possible option for tuning the weights w_{ij} is to find a global optimum over all contributing platforms. This, however, is rather difficult especially in the case of many interacting platforms. Therefore a multi-agent approach has been preferred. The determination of the weights is done by producer-consumer agent pairs in a MB scenario that is presented in the following.

Assume that to every local system S_i (platform) belongs a set of m producer agents Pag_{ij} and m consumer agents Cag_{ij}. Producer and consumer agents sell and buy, respectively, the weights w_{ij} on the basis of a common price p_i. Producer agents Pag_{ij} supply weights w_{ij_p} and try to maximize specific local profit functions ρ_{ij} where "local" means "belonging to system S_i". On the other hand, consumer agents Cag_{ij} demand for weights w_{ij_c} from the producer agents and try to maximize specific local utility functions U_{ij}. The whole "economy" is in equilibrium as the sum over all supplied weights w_{ij_p} is equal to the sum over all utilized weights w_{ij_c} .

$$\sum_{j=1}^{m} w_{ij_p}(p_i) = \sum_{j=1}^{m} w_{ij_c}(p_i) \quad (11)$$

A 'trade' between a producer and consumer agent is managed by cost functions for both types of agents. We define a local utility function for the consumer agent Cag_{ij}

$$Utility = benefit - expenditure$$
$$U_{ij} = \tilde{b}_{ij} w_{ij_c} - \tilde{c}_{ij} p_i (w_{ij_c})^2 \quad (12)$$

where $\tilde{b}_{ij}, \tilde{c}_{ij} \geq 0$, $p_i \geq 0$. Furthermore a local profit function is defined for the producer agent Pag_{ij}

$$profit = income - costs$$
$$\rho_{ij} = g_{ij} p_i (w_{ij_p}) - e_{ij} (w_{ij_p})^2 \quad (13)$$

where $g_{ij}, e_{ij} \geq 0$ are free parameters which determine the average price level. It has to be stressed that both cost functions (12) and (13) use the same price p_i on the basis of which the weights w_{ij} are calculated.

From the system equation (10) we define further a local energy function to be minimized

$$\tilde{J}_{ij} = v_{d_i}^T v_{d_i}$$
$$= a_{ij} + b_{ij} w_{ij} + c_{ij} (w_{ij})^2 \rightarrow min \quad (14)$$

where $\tilde{J}_{ij} \geq 0, a_{ij}, c_{ij} > 0$.

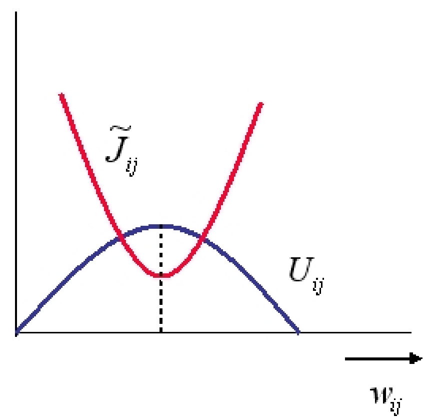

Fig. 6. Energy and utility function

The question is how to combine the local energy function (14) and the utility function (12) , and how are the parameters in (12) to be chosen? An intuitive choice

$$\tilde{b}_{ij} = |b_{ij}|, \quad \tilde{c}_{ij} = c_{ij} \quad (15)$$

guarantees $w_{ij} \geq 0$. It can also be shown that, independently of a_{ij}, near the equilibrium $v_{d_i} = 0$, and for $p_i = 1$, the energy function (14) reaches its minimum, and the utility function (12) its maximum, respectively (see Fig. 6).

With (15) the utility function (12) becomes

$$U_{ij} = |b_{ij}| w_{ij_c} - c_{ij} p_i (w_{ij_c})^2 \quad (16)$$

Maximization of the local utility function (16) leads to

$$w_{ij_c} = \frac{|b_{ij}|}{2c_{ij}} \cdot \frac{1}{p_i} \quad (17)$$

Maximization of the local profit function (13) yields

$$w_{ij_p} = \frac{p_i}{2\eta_{ij}} \quad where \quad \eta_{ij} = \frac{e_{ij}}{g_{ij}} \quad (18)$$

Substituting (17) and (18) into (11) gives the prices p_i for the weights w_{ij_p}

$$p_i = \sqrt{\frac{\sum_{j=1}^{m} |b_{ij}|/c_{ij}}{\sum_{j=1}^{m} 1/\eta_{ij}}} \tag{19}$$

Substituting (19) into (17) yields the final weights w_{ij} to be implemented in each local system. Once the new weights w_{ij} are calculated, each of them has to be normalized with respect to $\sum_{j=1}^{m} w_{ij}$ which guarantees the above requirement $\sum_{j=1}^{m} w_{ij} = 1$.

V. MB OPTIMIZATION OF OBSTACLE AVOIDANCE

A. MBO between active mobile platforms

In the following the optimization of obstacle avoidance between moving platforms by MB methods will be addressed. Coming back to the equation of the system of mobile robots (10)

$$v_{d_i} = vo_i + \sum_{j=1, i\neq j}^{m_p} w_{ij} v_{ij_p} \tag{20}$$

where vo_i is a subset of the RHS of (5) - a combination of different velocities (tracking velocity, control terms, etc.), v_{ij_p} reflects the repelling forces between platforms P_i and P_j. The *global* energy function (14) reads

$$\begin{aligned} \tilde{J}_i &= vo_i^T vo_i + 2vo_i^T \sum_{j=1,i\neq j}^{m_p} w_{ij} v_{ij_p} \\ &+ (\sum_{j=1,i\neq j}^{m_p} w_{ij} v_{ij_p})^T (\sum_{j=1,i\neq j}^{m_p} w_{ij} v_{ip}) \end{aligned} \tag{21}$$

The *local* energy funcion reflects only the energy of a pair of two interacting platforms P_i and P_j

$$\begin{aligned} \tilde{J}_{ij} &= vo_i^T vo_i + (\sum_{k=1,k\neq i,j}^{m_p} w_{ik} v_{ik_p})^T (\sum_{k=1,k\neq i,j}^{m_p} w_{ik} v_{ik_p}) \\ &+ 2 \sum_{k=1,k\neq i,j}^{m_p} w_{ik} vo_i^T v_{ik_p} \\ &+ 2w_{ij}(vo_i^T + \sum_{k=1,k\neq i,j}^{m_p} w_{ik} v_{ik_p}^T) v_{ij_p} \\ &+ w_{ij}^2 (v_{ij_p}^T v_{ij_p}) \end{aligned} \tag{22}$$

Comparison of (22) and (14) yields

$$\begin{aligned} b_{ij} &= 2(vo_i^T + \sum_{k=1,k\neq i,j}^{m_p} w_{ik} v_{ik_p}^T) v_{ij_p} \\ c_{ij} &= (v_{ij_p}^T v_{ij_p}) \end{aligned} \tag{23}$$

while neglecting a_{ij} because a_{ij} does not contribute to the MBO process.

B. MBO between active mobile platforms and passive obstacles

In this subsection the MBO between platforms will be extended by the MBO between a mobile platform P_i and several obstacles O_j ($j = 1...m_{ob}$). The motivation for this is that with the usual potential fields the platforms move normally as close as possible around the obstacles which might be an undesired behavior. Sometimes it would be better if the platform would navigate in a more conservative way so that there remains always an area around the platforms giving more space for additional unforseen maneuvers. In the last subsection, the MBO of repulsive velocities between the platforms has been described, whereas each involved agent (platform) is able to react actively. However, considering MBO between active platforms and passive obstacles active reactions from the obstacles cannot be expected. Therefore the MBO approach has to be adapted to the application to passive obstacles. In the following the optimization of obstacle avoidance by MBO methods between platforms on the one hand and between platforms and passive obstacles on the other hand will be addressed. Splitting up repulsive velocities between platforms v_{ij_p} on the one hand and between platforms and passive obstacles $v_{ij_{ob}}$ on the other hand leads to the equation of the system of mobile robots plus passive obstacles

$$v_{d_i} = vo_i + \sum_{j=1,i\neq j}^{m_p} w_{ij} v_{ij_p} + \sum_{j=1}^{m_{ob}} w_{ij_{ob}} v_{ij_{ob}} \tag{24}$$

where

- vo_i - subset of the RHS of (5), a combination of different velocities (tracking velocity, control terms, etc.)
- $w_{ij_{ob}}$ - weights for repulsive velocities $v_{ij_{ob}}$.

The difference between (24) and (20) is that in (20) the repulsive velocities between platforms and obstacles $v_{ij_{ob}}$ are included in vo_i whereas in (24) $v_{ij_{ob}}$ appear explicitely. For $w_{ij_{ob}} = 1$ the results of (24) and (20) are the same which is however not the case for $w_{ij_{ob}} \neq 1$.

Then the *global* energy function (14) reads

$$\begin{aligned} \tilde{J}_i &= vo_i^T vo_i + 2vo_i^T (\sum_{j=1,i\neq j}^{m_p} w_{ij} v_{ij_p} + \sum_{j=1}^{m_{ob}} w_{ij_{ob}} v_{ij_{ob}}) \\ &+ 2(\sum_{j=1,i\neq j}^{m_p} w_{ij} v_{ij_p})^T (\sum_{j=1}^{m_{ob}} w_{ij_{ob}} v_{ij_{ob}}) \\ &+ (\sum_{j=1,i\neq j}^{m_p} w_{ij} v_{ij_p})^T (\sum_{j=1,i\neq j}^{m_p} w_{ij} v_{ip}) \\ &+ (\sum_{j=1}^{m_{ob}} w_{ij_{ob}} v_{ij_{ob}}))^T (\sum_{j=1}^{m_{ob}} w_{ij_{ob}} v_{ij_{ob}}) \end{aligned} \tag{25}$$

The *local* energy funcion reflects only the energy of a pair of two interacting platforms P_i and P_j

$$
\begin{aligned}
\tilde{J}_{ij} &= vo_i^T vo_i + (\sum_{k=1,k\neq i,j}^{m_p} w_{ik} v_{ik_p})^T (\sum_{k=1,k\neq i,j}^{m_p} w_{ik} v_{ik_p}) \\
&+ (\sum_{k=1}^{m_{ob}} w_{ik_{ob}} v_{ik_{ob}})^T (\sum_{k=1}^{m_{ob}} w_{ik_{ob}} v_{ik_{ob}}) \\
&+ 2vo_i^T (\sum_{k=1,k\neq i,j}^{m_p} w_{ik} v_{ik_p} + \sum_{k=1}^{m_{ob}} w_{ik_{ob}} v_{ik_{ob}}) \qquad (26) \\
&+ 2w_{ij} {v_{ij}}_p^T (vo_i + \sum_{k=1,k\neq i,j}^{m_p} w_{ik} {v_{ik}}_p^T + \sum_{k=1}^{m_{ob}} w_{ik_{ob}} v_{ik_{ob}}) \\
&+ w_{ij}^2 ({v_{ij}}_p^T {v_{ij}}_p)
\end{aligned}
$$

Comparison of (26) with (14 yields

$$
\begin{aligned}
b_{ij} &= 2{v_{ij}}_p^T (vo_i + \sum_{k=1,k\neq i,j}^{m_p} w_{ik} v_{ik_p} + \sum_{k=1}^{m_{ob}} w_{ik_{ob}} v_{ik_{ob}}) \\
c_{ij} &= {v_{ij}}_p^T {v_{ij}}_p \qquad (27)
\end{aligned}
$$

The *local* energy funcion considers only the energy of one platform P_i with respect to obstacle O_l

$$
\begin{aligned}
\tilde{J}_{il} &= v_{d_i}^T v_{d_i} \\
&= vo_i^T vo_i + (\sum_{k=1,k\neq i}^{m_p} w_{ik} v_{ik_p})^T (\sum_{k=1,k\neq i}^{m_p} w_{ik} v_{ik_p}) \\
&+ (\sum_{k=1,k\neq l}^{m_{ob}} w_{ik_{ob}} v_{ik_{ob}})^T (\sum_{k=1,k\neq l}^{m_{ob}} w_{ik_{ob}} v_{ik_{ob}}) \\
&+ 2vo_i^T (\sum_{k=1,k\neq i}^{m_p} w_{ik} v_{ik_p} + \sum_{k=1,k\neq l}^{m_{ob}} w_{ik_{ob}} v_{ik_{ob}}) \qquad (28) \\
&+ 2(\sum_{k=1,k\neq i}^{m_p} w_{ik} v_{ik_p})^T (\sum_{k=1,k\neq l}^{r} w_{ik_{ob}} v_{ik_{ob}}) \\
&+ 2w_{il_{ob}} v_{il_{ob}}^T (vo_i + \sum_{k=1,k\neq i}^{m_p} w_{ik} v_{ik_p} + \sum_{k=1,k\neq l}^{m_{ob}} w_{ik_{ob}} v_{ik_{ob}}) \\
&+ w_{il}^2 v_{il_{ob}}^T v_{il_{ob}}
\end{aligned}
$$

Comparison of (28) and (14 yields

$$
\begin{aligned}
b_{il} &= 2v_{il_{ob}}^T (vo_i + \sum_{k=1,k\neq i}^{m_p} w_{ik} v_{ik_p} + \sum_{k=1,k\neq l}^{m_{ob}} w_{ik_{ob}} v_{ik_{ob}}) \\
c_{il} &= v_{il_{ob}}^T v_{il_{ob}} \qquad (29)
\end{aligned}
$$

Here one has to mention some exception when dealing with weights ${w_{ij}}_{ob}$ for the repulsive velocities ${v_{ij}}_{ob}$ of the objects: If one would use the computation of weights as before, then weights of the repulsive velocity of an object could appear to be much lower than *1*. This would possibly lead to a strong weakening of potential fields resulting in collisions between platforms and obstacles since obstacles cannot actively avoid. Therefore the weight resulting from MBO is changed into its 'negation'

$$\tilde{w_{ij}}_{ob} = C_{ob}(1 - {w_{ij}}_{ob}) \qquad (30)$$

where C_{ob} is a positive design parameter. The simulation shows the practicability of the method.

VI. Simulation results

The following simulation results consider mainly the obstacle avoidance of a multi-robot system (restricted to 3 platforms without loss of generality) in a relatively small area. The sensor cone of a platform amounts to +/- 170. Inside the cone a platform can see another platform within the range of 0-140 units. The platforms P_1 and P_3 are approaching head-on. At the same time platform P_2 crosses the course of P_1 and P_3 a right before their avoidance maneuver. If there were only platforms P_1 and P_3 involved, the avoidance maneuver would work without problems. According to the built-in traffic rules both platforms would move some steps to the right (seen from their local coordinate system) and keep heading to their target after their encounter. Platform P_2 works as a disturbance since both P_1 and P_3 react on the repulsive potential of P_2 which has an influence on their avoidance manoeuver. The result is a disturbed trajectory (see Fig. 7) characterized by drastic changes especially of the course of P_3 during the rendezvous situation. A collision between P_1 and P_3 cannot be excluded because of the crossing of the courses of P_1 and P_3. This also shows up in the plot Fig. 8, (subplot B31, timestep 190) where we notice quite high repelling forces between platforms P_2 and P_3 causing in turn high avoidance movements.

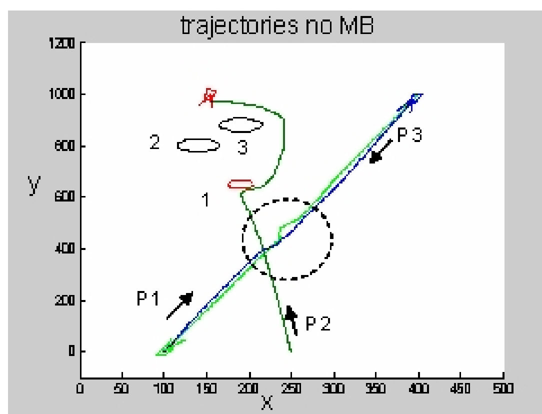

Fig. 7. Approach, no MB optimization

When activating the MBO, we obtain a behavior that follows both the repulsive potential law for obstacle avoidance and the traffic rules during approaching head-on (see Fig. 9). There is no crossing of tracks between P_1 and P_3 any more which comes from the MB optimization of the repelling forces between platforms P_1, P_2, and P_3 and a respective tuning of the weights w_{ij}. Figure 10 shows the resulting weights. We also notice that w_{12} and w_{13}, w_{21} and w_{23}, and w_{31} and w_{32} are pairwise mirror-inverted due to the condition

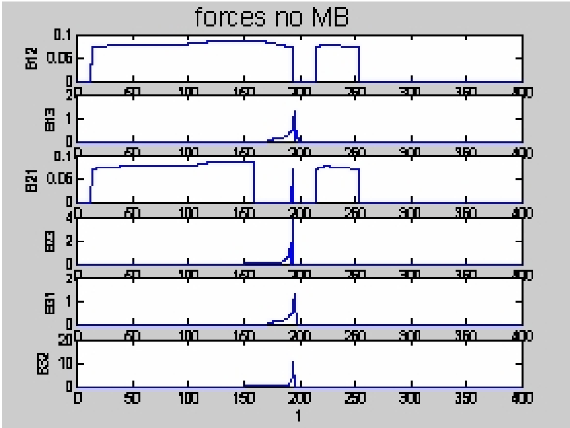

Fig. 8. forces, no MB

$\sum_{j=1,i\neq j}^{m} w_{ij} = 1$ (see also eq. (10)). Since the platforms hold a certain distance from each other, the repelling forces between the platforms are lower than without MBO (see Fig. 11), (subplot B31, timestep 190).

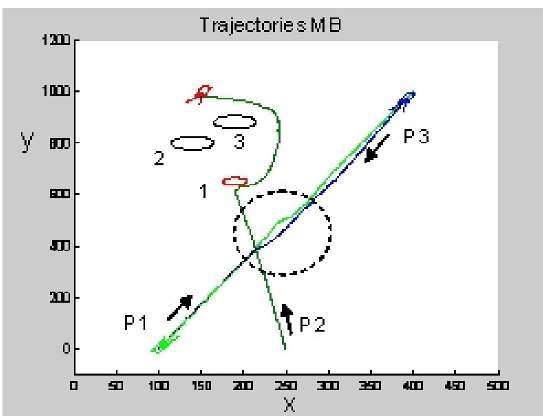

Fig. 9. Approach, with MB optimization

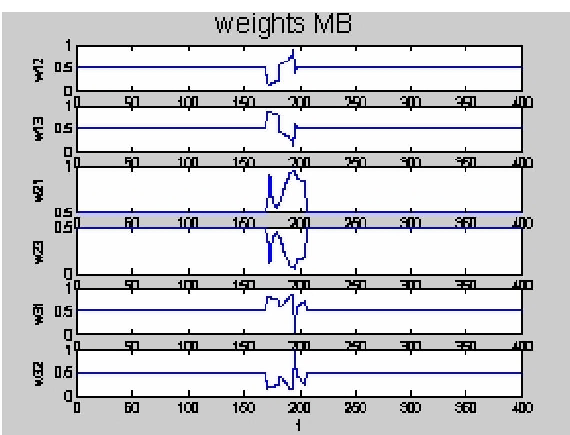

Fig. 10. weights, with MB

In further simulations the platforms are required to move on circles with different speeds, similar diameters and center points while avoiding other platforms and static obstacles on

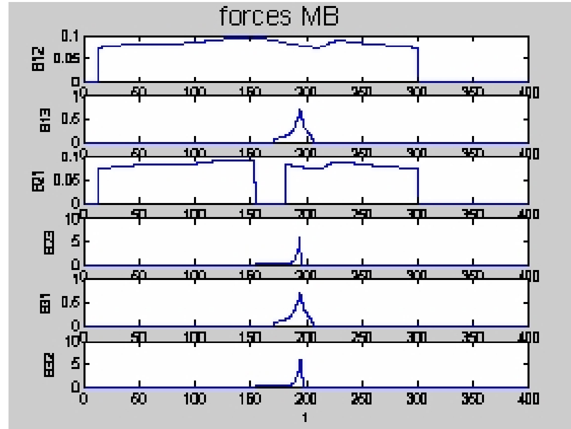

Fig. 11. forces, with MB

their tracks. To determine the smoothness of the trajectories, the averages of the curvatures along the trajectories of the platforms were calculated. Figures 12 and 13 show the actual corrected trajectories of the platforms where the circular reference trajectories are not explicitly shown. It turned out that the use of MBO leads to a significant improvement of the smoothness of trajectories.

Furthermore, the influence of MBO on the behavior between platforms and obstacles can be shown by an example depicted in Figs. 14 and 15 where platform P_2 passes the obstacles in a much larger distance if MBO is switched on.

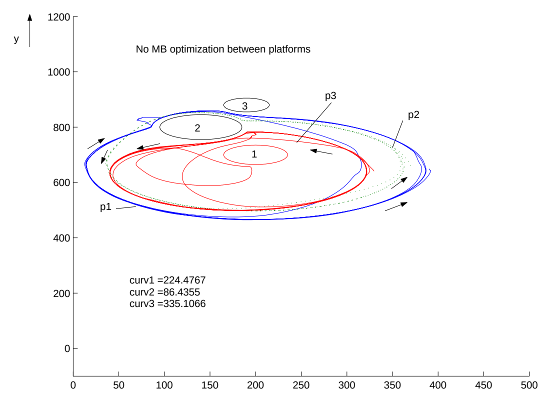

Fig. 12. moving on circles, no MB

VII. Conclusions

Navigation and obstacle avoidance of mobile robots can be performed by a variety of principles like artificial potential fields, traffic rules, and control methods. It has also been shown that a 'deformation' of central symmetry by using fuzzy rules may be helpful because it takes better the robot-object scenario into account. An important aspect is the market-based optimization (MBO) of competing potential fields of mobile platforms. MBO imitates economical behavior and the competition between consumer and producer agents. By means of MBO some potential fields will be strengthened and some weakened depending on the actual scenario. This

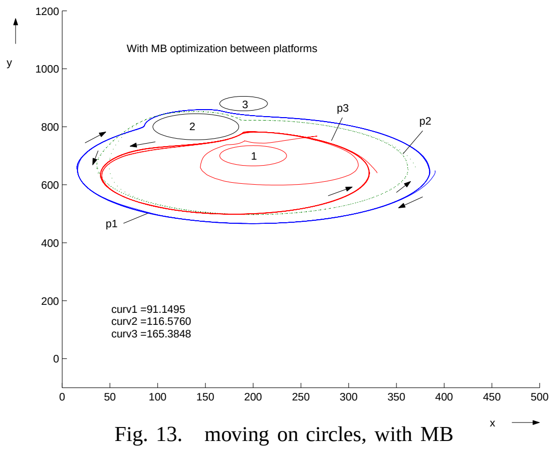

Fig. 13. moving on circles, with MB

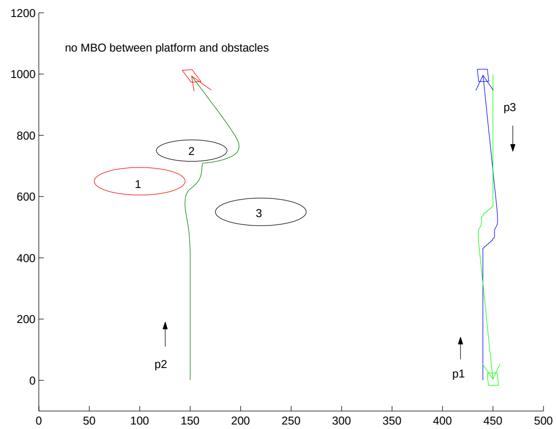

Fig. 14. No MBO between platforms and obstacles

is required when more than two robots compete within a small area which makes a certain minimum distance between the robots and appropriate maneuvers necessary. Therefore, MB navigation allows smooth motions in such situations. Simulation experiments with simplified robot kinematics and dynamics have shown the feasibility of the presented method. A future aspect of this work is the implementation of the

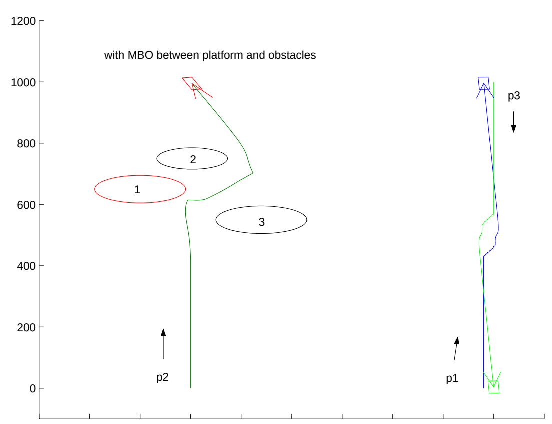

Fig. 15. With MBO between platforms and obstacles

algorithm on a set of real mobile robots.

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