MATHEMATICAL LEARNING PROCESSES SUPPORTED BY AUGMENTED REALITY

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The authors of this paper are involved in an ongoing project with the aim of investigating ICT-supported activities for the learning of mathematics where real-world images are mixed with computer-generated 3D images. The present paper explores the ways in which four students (15 years old) try to make sense of a task that calls for reflection on the concept of scale. The analysis shows how this specific kind of learning activity can challenge students to vary and coordinate among representations offered within the activity, thereby creating opportunities to extend and strengthen their networks of knowledge elements associated with the current learning object.

INTRODUCTION

An ICT-supported setting may offer interesting possibilities for the learning of mathematics because it can, and often does, provide students with several representations with which they can work (Parnafes & diSessa, 2004). Further variation of representations may be offered by allowing for students to make use of real objects in the virtual setting, thereby providing opportunity for students to interact physically with the virtual objects and also providing tangible feedback (Scarlatos, 2006).

The present paper is part of an ongoing project regarding design of ICT-supported learning activities, which are developed in collaboration between researchers in mathematics education, researchers/developers in media technology and high school teachers. A central aspect in this work is to investigate the use of *augmented reality (AR)*, a technology that allows for mixing real-world images with computer-generated images (Milgram & Kishino, 1994). Although Sutherland already in the 1960's (1965) developed the first AR interface, it is only recently that researchers have explored its potential uses for formal education (Zhou et al., 2008).

Cobb (2007) formulates a mission for research in mathematics education as striving to develop, test and revise hypothetical learning activities, which are designed in order to support envisioned learning processes of a learning object. In the present case, the envisioned learning processes relate to a metaphor of considering students' understanding of a learning object on the form of a network of knowledge elements (Hiebert & Carpenter, 1992). On the basis of this metaphor, our learning activity aims at extending and strengthening students' ability to use and integrate aspects of the *concept of scale,* by varying and coordinating among representations offered within an explorative AR-supported setting. More specifically, the aim of the present paper

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is to explore the ways in which students interpret and try to make sense of a task that calls for reflection on the concept of scale in a specifically designed learning activity in which the students are offered opportunities to vary and link between real and virtual sources of references.

OVERALL METHODOLOGICAL CONSIDERATIONS

Our research project relies to a great extent on the combined competencies within the development team. Such a methodology is in line with a co-design approach (Roschelle & Penuel, 2006) that shares similar affinities with *Design experiments* (Cobb et al., 2003). Co-design may be defined as a highly facilitated team-based process in which teachers, researchers, and developers work together to design an educational innovation (Penuel et al., 2007). The methodology of Design experiments (DE) provides a structure for design-based research according to five principles. The first principle of DE, *develop theories*, is followed by *control*: "The intent is to investigate the possibilities for educational improvement by bringing about new forms of learning in order to study them" (Cobb et al., 2003, p. 10). Control implies the need for *prospective* and *reflective analyses*. The prospective and reflective aspects come together in a fourth characteristic of DE, *iterative design*. Iterations are carried out with the aim at enhancing learners' thinking and reasoning in relation to the hypothetical learning activity. The fifth principle refers to the *pragmatic roots* of DE. As schoolteachers take active part in the design process, we feel confident that the designed activities are relevant for teachers' practice.

In this paper, we put focus on the reflective analysis. As a background, we present the task and its setting and briefly discuss the prospective analysis which has been elaborated previously (Sollervall, Nilsson & Spikol, in press). The reflective analysis is based on observations of students' actions, while the prospective analysis was based on hypothetical actions. The analytical construct of contextualization will be used to guide and control our reflective analysis.

ANALYTICAL CONSTRUCT OF CONTEXTUALIZATION

Our research is guided by a knowledge metaphor where understanding may be viewed as a network of coordinated knowledge elements. The analytical construct of *contextualization* offers a tool to organize an analysis of such networking. The construct of contextualization makes us alert to the different aspects of a learning object that learners attend to during a learning activity, with focus on the different resources that they use for their exploration of and communication about that learning object (Nilsson, 2009).

Context is often used to describe the physical and discursive setting in which a learning activity takes place (Säljö, 2000). This is not the case in the present approach. To speak about students' processes of contextualization is to speak about how learners struggle to render a phenomenon or concept meaningful in personal, cognitive contexts of interpretation (Caravita & Halldén, 1994). In order to clarify how different contextual elements may interact and serve as points of reference in the learners' processes of contextualization, we can delineate at least three types of analytical contexts (Halldén, 1999). The *conceptual* context refers to personal constructions related to concepts and subject matter-structures. An explorative learning activity particularly highlights the *situational* context, which refers to interpretations made in the interaction between the individual and the immediate surroundings, including interpretations of figurative material, possible actions and directly observable sensations. Then there is the *cultural* context, which refers to constructions of discursive rules, conventions, patterns of behavior, and other social aspects of the environment (Halldén, 1999).

SETTING THE SCENE

The technical setup includes an ordinary computer with webcam, a projector and a vertical board. In addition, software is needed to support the technique of augmented reality. The physical setting of the activity may be described as follows. A satellite photograph of a landscape, with a uniform length scale, is laid down horizontally on a table. A movable small tag (cardboard two-dimensional bar code) is placed on the photograph. For our analytical purposes we refer to this situational sub-context as the *photograph context* (Figure 1a).

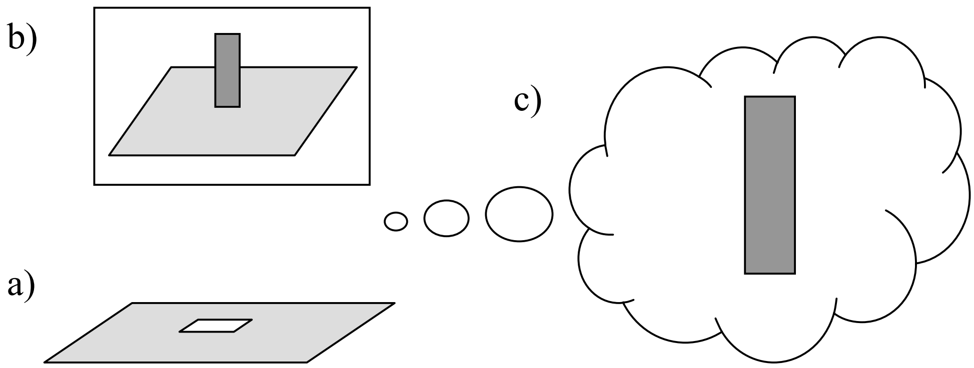

Fig 1. a) Tag on top of a photograph. Building can not be seen, just imagined. b) Projection of the photograph, showing a building on top of the tag in 3D. c) Building imagined in reality.

A webcam projects a picture of the photograph on the vertical board through a projector. The webcam is placed so that the length scale differs by (at least) a factor of 2 from front to back. When the webcam 'reads' the tag, the software supports showing a building (or any kind of picture) on top of the tag on the board. The 'tag-picture' (building) is shown virtually in three dimensions. The picture shown on the vertical board frames a situational sub-context which we call the *board context* (Figure 1b).

The choice of scale was discussed within the development team and was decided to be 1:800. Four Swedish students, all 15 years old and from the same class, were recruited to participate in the study. A satellite photograph of their hometown, measuring 70 cm x 100 cm, was prepared and was taped down on a table, which was placed 1.5 meters directly in front of the vertical board. The aforementioned tags were prepared to show buildings that were known to the students, including the Turning torso (in Malmö, Sweden), the Pyramid of Cheops, the Eiffel tower and the Cathedral of their town. Using buildings that were known to the students was assumed to be beneficial for their ability to evaluate whether their reasoning or a proposed solution seemed plausible.

The group of students included one girl, Alice, and three boys, Ralph, Edward and Larry (the names of the students have been changed). The students were used to working together in school in subjects other than mathematics. In addition they had already covered the concept of scale in school.

The session started with our technical developer giving the four students a short presentation of how the equipment worked. The task was communicated through written instructions, including information on how the tags worked. The students were simply asked to figure out the building's height in *reality* (the third situational sub-context of our analytical approach). They were free to use tools and take measurements both on the board and on the photograph, but this was not explicitly communicated. A whiteboard pen, a ruler and a calculator were available for them to use. The students were told that they could consult Håkan (one of the authors of this paper) during the activity if they had any questions.

What mathematical concepts, methods and strategies may the students bring to use in their personal contexts for interpreting a building's height in reality?[1]. The students may place a ruler or an object raised vertically from the table and look at the board to measure the height of the building in the scale of the photograph, or, they may measure the height of the building on the board and compare with a corresponding horizontal measurement on the board that may be related to a corresponding measurement on the photograph. Once the students have either of these measurements they may find the corresponding measurements for the real building by using either the concept of scale or by comparing with objects either in the photograph or on the board, such as cars or people, together with principles of similarity (proportions, Regula de Tri).

RESULTS AND ANALYSIS

The students spent 30 minutes working on the task. The session was videotaped and additionally audio recorded. During the activity, it was mainly Alice and Ralph that took turns in the group work. Larry was more or less quiet during the whole session. Edward contributed by moving the tags, mostly on Alice or Ralph's initiatives.

[1] Further details about the mathematical intentions behind the design are given in Sollervall, Nilsson and Spikol (in press), where the prospective analysis was the object of investigation.

Linking measurements from the board context to the real world

The group has chosen to figure out the height of the Turning torso and Alice takes the first initiative. Alice makes the group aware that the scale of the photograph says 1:800. Then she points to the board and asks the group about the height of the image. On Ralph's suggestion, Alice takes a ruler, 50 cm in length, and starts to measure the height of the building on the board.

Alice:	[Measuring the height]…well…
Ralph:	39?
Alice:	Yes, 39.
Ralph:	[writing down 39 on a paper] Multiply with 800 then?
Group:	Yes.

Ralph asks for a calculator. However, while he waits, he finishes the calculation with a paper and pencil algorithm.

Ralph:	…then it will be….
Alice:	31200.
Ralph:	It is not that high!
Alice:	Is it not?
Ralph:	No, it can't be 3000 meter high.
Alice:	[Laugh] think about an airplane…
Edward:	Yes, but it is centimeters.

The group knows that there exist no buildings of 3000 meters and they realize the absurdness in the estimation. Their reasoning, emerging in the frame of augmented reality, is coordinated and tested against what is known and seems to be plausible in the real world. This testing gives incitement for reflection, supporting the students to adjust Ralph's calculation and to agree on an estimation of the height, which better matches their understanding of heights of buildings in the real world.

The emergence and reduction of conflicting views

The group comes to an agreement of 312 m for the height of the building. The students do not have access to the exact height of the building, but the estimation appears plausible for them. They ask Håkan if it is correct. Håkan does not respond to the question explicitly. Instead, he challenges the students by implicitly questioning whether the scale works on the board.

Håkan:	We can test something here. If we look at it here [drags the tag away from the webcam] and then place it here [drags the tag closer to the webcam], then it [the 3D image] becomes larger.
Ralph:	Yes, then it will not be the same.

Håkan: Isn't that strange? Then it will be higher when you measure on the board, or? [moving the tag away from the camera so that the size of the building is reduced on the board].

Already the fact that Håkan does not, in a direct way, confirm the students' solution make them doubt whether they have solved the task in a proper manner or not (cultural context). When Håkan moves the tag it becomes obvious to the students, and particularly to Ralph, that there are flaws in their reasoning.

Alice suggests that they should have something on the photograph to compare the building with. Edward, who has been noting that the building may be laid down on the photograph if the tag is tilted towards the webcam, asks if this feature of the equipment could be used. The group makes a short effort to explore his initiative. In doing this, they agree on that the picture of the building has to appear horizontally on the board in order to provide an appropriate measure to use for calculations. However, when Edward tries to place the building horizontally, the tag loses contact with the webcam and the building disappears.

After a moment of silence Alice continues:

Alice: But, if one tries to measure something here on the map and then compares that to…

Ralph: Yes, but, there is nothing on the map that is marked down, it is just… it is 3D.

None in the group are able to suggest how they may proceed. They are silent for about 7 seconds and seem discouraged. Håkan chooses to interfere and shows the group how the webcam can project the ruler in an upright position on the board. On the table only the ruler can be seen, but on the board the ruler is shown together with the building. On account of this intervention, Ralph asks Edward to drag the tag to different locations on the photograph. Ralph follows Edward's moves and places the ruler beside the tag. The group discovers that the building's height always stays at 24 centimeters on the ruler, regardless of where the comparison is made. Now the ruler is used in the photograph context, while the students' reasoning requires that the photograph context is coordinated with the board context where the measurement 24 cm may be read off. Ralph calculates the new height to 192 meters by multiplying 24 with 800 and transforming to meters. The height of 192 meters seems plausible to the students and Håkan also confirms this by acknowledging their solution[2]. The group then continues to measure some other buildings, using the same solution strategy.

After the students leave the room, their teacher comments that the students should do more of this kind of activities to get used to working on own initiatives and develop confidence to explore different strategies.

[2] The exact height of the Turning torso is 190 (190.4) meters.

DISCUSSION AND FUTURE EFFORTS

The present paper explores the possibilities to use the technique of augmented reality for designing learning activities. The design process is guided by a metaphor where learning and understanding of a mathematical concept are considered on the form of a network of connected knowledge elements and information about the concept in question (Hiebert & Carpenter, 1992).

The analysis shows how the students' understanding of a building's height in reality supported their processes of reflection, from which they were encouraged to re-organize their thinking. However, the analysis also indicates that the students' difficulties in solving the task can not only be considered as a conceptual problem. To be able to manage a learning activity by varying and coordinating among representations related to a learning object, the students also have to be familiar with how the technique works and its potential for exploring aspects of the learning object. Hence, a natural next step in the co-design process would be to dig deep into the relation between technical and conceptual issues in order to investigate how the technique of augmented reality may be used to support students' learning of mathematics.

The students were not used to solving tasks by exploring possible strategies by themselves and expected confirmation by external responses. During further iterations of the design it would be desirable to include scaffolding features (such as written hints, offering alternative pre-designed tasks with reduced complexity) that may stimulate students' own initiatives by reducing external interference.

The current task requires coordination between three contexts (photograph, board, and real world). From a research perspective, it would be valuable to further investigate the dynamics in the group's coordination between these contexts. On what grounds do they choose a specific context, change context or coordinate contexts? How do they draw on features of a specific context and how do they analyse the possibilities and limitations of that context?

As another future effort, we intend to try out the activities with younger students that have not yet (formally) encountered the concept of scale. These students may be expected to use both formal concepts (such as similarity, proportions) and informal concepts related to their everyday life while exploring the task. Such an investigation would implicitly shed light on possible ways to draw on their pre-conceptions to introduce the concept of scale to these students.

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